# ON THE FIELD EQUATIONS GOVERNING THE ORIGIN AND EVOLUTION OF THE SPACE AND TIME AND PARTICLES OF THIS UNIVERSE


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On the field equations governing the origin and evolution of compactified hyper-dimensional-energy universes



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Twentieth century science produced a clear need for a *universe theory* (UT) with an inherent capacity to explain how compactified hyper-dimensional universes are explosively created and sustained. *Space-energy theory,* in (R. Var, Foundations of Physics **5**, 3, (Sept. 75, pp. 407-431)), offered a embryonic (1 + 4)-dimensional candidate for such a theory. Here I derive a definitive $(1 + p \geq 4)$-dimensional UT called *hyper-energy theory*—as the third of three increasingly comprehensive gauge-field theories (GFT1−3) which are derived from the physical implications of a gravity theorem that Maxwell published, along with his electromagnetic field theory, in 1864; stating that: *Gravity is a mass-induced reduction of an enormous intrinsic energy density,* ${}^{0}\in_{m}$, *that characterizes the space-medium*. GFT1, called **w**−gauge theory, yields five ways that particles couple to $\in_{m} \leq {}^{0}\in_{m}$ via a single coupling-strength function; $\ell = 1/[1 - \mathbf{w}^2]^{1/2}$, of their propagation velocity, **w**, in order conserve their energy and momentum. GFT1 is shown to cover special relativity theory while introducing the following two revolutionary discoveries: a) The flow velocity **u** of $\in_{m}$ is a locally unobservable 3−vector-potential of the two previously disparate kinds of gravity referred to as matter-gravity and 'elevator−gravity'. b) The potential of Maxwellian-gravity, $\Phi_m = \frac{1}{2}\mathbf{u}^2$, provides fluid-$\in_{m}$ explanations for the potential $(-\Phi_n/c^2)$ of Newtonian gravity and for the black holes and gravitational red-shifts deduced from general relativity theory. GFT2 is a (1 + 3) tensor generalization of **w**-gauge theory precipitated from Einstein's *overly general* relativity theory by employing $\ell$ to give the 4-scalar differential, ds, a specific practical form, $dx^0/\ell$, which causes the resulting theory—called Einstein-Maxwell (EM) gravity theory—to be harmonious with **w**−gauge theory and thus Maxwell's gravity theorem. The interactions of EM gravity and particles are then evaluated in sufficient depth to show that EM gravity is a readily quantizable solution of the long standing quantum-gravity problem. Hyper-energy theory (GFT3) is then logically deduced as being—to a first approximation—nothing more than, and nothing less than, a (p−3)-dimensional extension of the (1 + 3)-dimensional laws governing the dynamics of an abstractly continuous (non particulate) medium of compressible and inviscid mass-energy. I then demonstrate the $e^{(p-3)}$ proportional efficacy of hyper-energy theory with multi big-bang driven, $p$−invariant, qualitative solutions of the hyper energy field equations that can be seen to account for: a) Compactification and ${}^{0}\in_{m}$ structure—with $(p - 3)$ locally orthogonal (flat) time-flow-sourced hyper-dimensions. b) The Inflationary and Hubble expansion phases. c) The unifying role of a soliton Higgs-field in determining: 1) Cosmological particle-generation; 2) Maxwell's gravity theorem; 3) The quasi-(1 + 3)-dimensional propagation of particles and their de Broglie waves; 4) The elementary particle spectrum; And 5) The physical nature of both time and superstrings. Hence, a single, multi-component, super gauge-field representing *the flow of time*—which controls particle structures and interactions via its many and various types of physically comprehensible symmetry breakings—is accurately identified by hyper-energy theory as the $(1 + p)$-dimensional flux of hyper-energy through the propagationally expanding and solitonally compactified '3−space' of this universe.

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## I. INTRODUCTION

### A. The Gravity Theorem of James Clerk Maxwell.

In his classic 1864 paper "A Dynamical Theory of the Electromagnetic Field" Maxwell also described his attempt to derive a $1/r^2$ theory of gravity that would be consistent with his newly formulated field equations of electrodynamics. That attempt led Maxwell to the following three conclusions:

a) The medium of 3-space is characterized by an enormous intrinsic energy.

b) Gravity is a mass-proportional reduction of that relatively enormous 3-space-energy.

c) "As I am unable to understand in what way a medium can possess such properties I cannot go further in this direction in searching for the cause of gravitation."[1]

I here raise Maxwell's conclusions (a) and (b) to the level of a theorem, called *Maxwell's gravity theorem*, and I assert that Maxwell's conclusion (c) posed a well defined challenge for the scientific community that is referred herein to as the *3-space-energy problem*.

I then assert that, in defining the *3-space-energy problem,* James Clerk Maxwell passed-on the challenge of explaining and exploiting the enormous classical energy of the 3-space medium to early 21[st] century theorists who now possess all of the mathematical tools, knowledge, and, experimental data, needed to do just that—*by simply employing Maxwell's gravity theorem as a driving physical principle.*

The truth of this assertion is then demonstrated in this paper by employing Maxwell's gravity theorem and contemporary scientific knowledge to derive the three increasingly comprehensive gauge-field theories (GFT1−3) summarized in the abstract.

The remaining sections of this paper are organized as follows:

Section II. Founding Principles, Equations, Terminologies, and Working Notations.

- A. New terminologies consistent with Maxwell's gravity theorem.
- B. Four interrelated working implications of Maxwell's gravity theorem.
- C. The founding theoretical concept and equation of hyper-energy theory.
- D. The pragmatically essential but strictly complementary nature of GFT1.
- E. Analytic constructs and variables for deriving GFT1, 2 and 3.

Section III. GFT1: A **W**-Gauge Theory of $\in_m$-Field-Particle Coupling—Covering special relativity and introducing Maxwellian gravity.

Section IV. GFT2: Einstein-Maxwell (EM) Gravity—Directly from the founding equation of hyper-energy theory derived in II.C.

Section V. Applications Demonstrating That EM Gravity Solves The Quantum-Gravity Problem.

Section VI. GFT3: The General Principles and Field Equations of Hyper-Energy Theory.

Section VII. On The Hyper-Energy Origin, Dynamics, and Structure of the ${}^0\in_m$–Continuum.

Section VIII. On The Hyper-Energy Origins of Massive and Massless $\delta\mathcal{E}_m$ Particles.

Section IX. On The Hyper-Energy Physics of Electric Charge and $\delta\mathcal{E}_m$ Energy, Inertia, and Gravity.

Section X. On Tapping the Flow of Time for Low Pollution Hyper-Energy Power and Propulsion.

Section XI. Some General Remarks concerning a Hyper-Energy Theory Completion Program.

## II. Founding Principles, Equations, Terminologies, and Working Notations.

### A. New terminologies consistent with Maxwell's gravity theorem.

The MKS system of units is employed throughout, and terms like *3–space-energy* or the *3–space-energy continuum* are used to supplant traditional terms like *space* or *3-space*, *empty-space, physical space or physical 3-space*, the *void* or *vacuum* of the foregoing, the *medium of space*, and *the luminiferous ether or aether*—terms which failed to suggest anything even remotely resembling the enormously rich *energy-medium* deduced, originally it seems, by Maxwell.

The symbol ${}^{0}\mathcal{E}_{\mathrm{m}}$ (m for Maxwell) denotes the *3-space-energy* contained in a given 3-volume (V) of the undisturbed *3-space-energy continuum,* and ${}^{0}\in_{\mathrm{m}} = {}^{0}\mathcal{E}_{\mathrm{m}}/\mathrm{V}$ then denotes the corresponding 3-volume density of ${}^{0}\mathcal{E}_{\mathrm{m}}$.

Terms like *3-space-energy* and the *3-space-energy continuum* will always refer to the *undisturbed* regions characterized by ${}^{0}\in_{\mathrm{m}}$. Since *3-space-energy* is implied by the symbols ${}^{0}\mathcal{E}_{\mathrm{m}}$ and ${}^{0}\in_{\mathrm{m}}$, the *3-space-energy* continuum will also be referred to as the ${}^{0}\mathcal{E}_{\mathrm{m}}$*-continuum* or the ${}^{0}\in_{\mathrm{m}}$*-continuum*.
Logically, any explanation for the ${}^{0}\in_{\mathrm{m}}$–continuum which is consistent with the big bang scenario and which maintains the hallowed principle of energy conservation can only come from the physics of a more fundamental, $(1 + p \geq 4)$-dimensional-energy continuum.[a] And since this conclusion is rigorously vindicated herein by GFT1–3, it is useful to note the logical distinction that must be made and hereafter born in mind; between *this* [ephemeral ${}^{0}\mathcal{E}_{\mathrm{m}}$ ] *universe* and *the* [permanent hyper-energy] *universe*. Namely:

> *The universe* is a permanent $(1 + p)$-dimensional *hyper-energy-universe*, $U_{4+m}$, $(m = (p - 3))$, in which *this* (quasi-$U_4$) *universe* is sustained as an explosively created, 100% dynamically compactified and coupled, *internal soliton energy state* of $U_{4+m}$.

Since the readily quantizable EM gravity theory (derived in Section IV) furnishes a useful mathematical link between all of our present quasi-(1 + 3) knowledge and the new $(1 + p)$ energy physics of *the universe*, it may be helpful to keep this revolutionary view of *the universe* in mind while studying the derivations of the gauge-field theories (GFT1–3) which substantiate this new paradigm.

### B. Four interrelated working implications of Maxwell's gravity theorem.

The gauge-field theories (GFT1–3) are derived in a straight forward manner from the following four physically interrelated and mathematically qualifiable *working implications* of Maxwell's gravity theorem (i-iv), referred to herein as ${}^{0}\mathcal{E}_{\mathrm{m}}$-Implications (i–iv):

(i) Given that ${}^{0}\mathcal{E}_{\mathrm{m}}$ is defined as being 3-dimensionally *continuous* and possessing an *enormous intrinsic energy*—relative to matter—it follows that, 3-volume for 3-volume, the rest-energy of any particle represents a miniscule disturbance or perturbation of ${}^{0}\mathcal{E}_{\mathrm{m}}$. The symbol $\delta\mathcal{E}_{\mathrm{m}}$ will then be used to denote any such miniscule perturbation of ${}^{0}\mathcal{E}_{\mathrm{m}}$.

The *symbol* $\delta\mathcal{E}_{\mathrm{m}}$ will also be used as an adjective to occasionally emphasize this new *perturbational* perception of *matter particles and fields* by referring to them as $\delta\mathcal{E}_{\mathrm{m}}$ field-particles and $\delta\mathcal{E}_{\mathrm{m}}$ fields. This $\delta\mathcal{E}$ *aspect* of the new physics is emphasized more strongly by saying that *all* $\delta\mathcal{E}_{\mathrm{m}}$ *matter particles and fields have a 100% coupling to the* ${}^{0}\mathcal{E}_{\mathrm{m}}$*–continuum*.

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[a] The $(1 + p)$ ordering of *hyper energy* coordinates was chosen over the traditional (p + 1) ordering of *spacetime* coordinates for reasons of notational convenience which become important in Sections VI – XI.

(ii) Given (i), Newton's laws of motion, and Maxwell's equations, it follows that the physical structure of ${}^{0}\mathcal{E}_{m}$ provides all $\delta\mathcal{E}_{m}$ field-particles with a common *propagational* mechanism for their transport through the 3−space-energy continuum—which is basically *frictionless*. The symbol $\mathbf{w} = w^{\alpha}\mathbf{e}_{\alpha}$ will then be used to denote the lightspeed normalized propagation velocity of an arbitrary $\delta\mathcal{E}_{m}$ particle or field point.

(iii) Given (ii), the electromagnetic basis for light and its constant propagation velocity (c) within ${}^{0}\mathcal{E}_{m}$ (as disclosed by Maxwell in his classic 1864 paper), plus some theoretical insights drawn from compressible hydrodynamics, I assume that $c^2$ is a proportionate measure of ${}^{0}\in_{m}$. The physical properties of Maxwell's ${}^{0}\in_{m}$-continuum can thus be said to be characterized by $c^2$, or equivalently, by either $\mathbf{w}^2 = 1$ or the null 3-scalar, $s^0$, where

$$(s^0)^2 \equiv (1 - \mathbf{w}^2) = 0. \tag{2.1}$$

(iv) Since $\delta\mathcal{E}_{m}$ *propagation* in $\in_{m}$ and a *relative flow field* of $\in_{m}$ with lightspeed normalized flow velocity, $\mathbf{u} = u^{\alpha}\mathbf{e}_{\alpha} = -\,\mathbf{w}$, are *complementary physical properties of continuum physics*, I assume that a more general flow field of $\in_{m} \leq {}^{0}\in_{m}$ exists relative to a massive $\delta\mathcal{E}_{m}$ particle—associated with the 3−dimensionally extended field-structure of the particle. Maxwell's gravity theorem therefore precipitates the existence of a new 3-vector field in physical theory,

$$\mathbf{p}_{m}(x^i) = \in_{m}(x^i)\, \mathbf{u}(x^i), \tag{2.2}$$

which, to a first approximation, describes a momentum density or flux of the enormous energy characterizing the medium of our compactified 3-space.

The field $\mathbf{p}_{m}(x^i)$ (m for Maxwell) is herein shown to function as a *3−vector potential of Maxwellian gravity.* Following the derivation of GFT1 and proof that GFT1 covers special relativity theory, we show that variations of $\mathbf{p}_{m}(x^i)$ described by $\partial\mathbf{p}_{m}/\partial x^0$ and $(\nabla\times\mathbf{p}_{m})$ have been routinely measured and exploited over the last half century for the practical purpose of inertial (${}^{0}\in_{m}$) navigation. Thereby revealing that the $\mathbf{p}_{m}(x^i)$ field already has the same degree of empirical vindication as the electromagnetic vector potential–if one merely takes gainful advantage of the liberty, allowed by GFT1—of describing the responses of *inertial sensors* in the same proper *in situ* manner as one typically describes the responses of various electromagnetic field sensors via their responses to temporal and spatial changes of the electromagnetic gauge fields, $\varphi_e$ and c**A**.

## C. The founding theoretical concepts and equation of hyper-energy theory.[2]

### 1. The founding theoretical concepts of hyper-energy theory.

The founding theoretical concepts (FTCs) of hyper-energy theory are just the two physical revelations of ${}^{0}\mathcal{E}_{m}$–implications (i–iv); suggesting that the 3-transport of *any* particle or field-point (p) is generally due to two distinct physical mechanisms: a) The *frictionless propagation* of p with lightspeed normalized 3-velocity **w** *relative to* its underlying $\in_{m}$-continuum. And b) The *frictionless flow* of $\in_{m}$ with lightspeed normalized 3-velocity **u** *relative to* the ${}^{0}\in_{m}$-continuum.

### 2. The founding equation of hyper-energy theory.

The founding equation of hyper-energy theory answers this simple question: If an arbitrary particle, p, is propagating with velocity **w** at a point where the $\in_{m}$-continuum is flowing with velocity **u**, what is p's velocity relative to the superficially undisturbed ${}^{0}\in_{m}$–continuum?

The answer:

$$d\mathbf{r}/dx^0 \equiv \beta(\,p) = \mathbf{u}(\in_m) + \mathbf{w}(p), \tag{2.3a}$$

therefore describes how the FTCs of hyper-energy theory serve to explain the *rectilinear velocity of p* referenced to the $^0\in_m$-continuum (and/or any given inertial observer, as we shall be demonstrating). Equation (2.3a) then quantifies a new parameter in physical theory referred to herein as the *generalized rectilinear-velocity* (GRV) of an arbitrary (massive or massless) $\delta\mathcal{E}_m$ particle or field point (p). The GRV and its magnitude $\beta$−defining the generalized rectilinear speed (GRS) of p−are mathematically clarified in Section II.D.7, and an angular complement of the GRV is described in Section II.D.8 with the help of the intervening analytic constructs.

*a. Historical note.*

In his valedictory Scientific American article of 1963, P. A. M. Dirac asserted that if a new field like $\mathbf{p}_m(x^i)$ could be successfully introduced into physical theory, it would lead to a *classical physics of the future* capable of modeling *particle structure*—unencumbered by the mathematical complications of the uncertainty relations which pertain only to *particle-interactions.*[3] As Dirac pointed out, to be successfully introduced it would have to be shown that $\mathbf{p}_m(x^i)$ preserves the *4−dimensional symmetry required by Einstein's relativity principle.* And as shown herein, this is precisely what **w-**gauge theory does.

## D. Analytic constructs and variables for deriving GFT1−3.

### 1. Point-like particles, compound particles, rigid bodies, and spacecraft laboratories.

A general $\delta\mathcal{E}_m$ field-point or point-like particle is represented by p. A *massless* p particle is represented by $p^0$. And a *massive* and *electrically uncharged* p particle is represented by $p^\pi$. $p^\pi$ particles carrying positive and negative electric charges are then denoted respectively by $_+p^\pi$ and $_-p^\pi$. A *compound, massive, electrically neutral* $\delta\mathcal{E}_m$ *particle*—consisting of two or more $p^\pi$ particles bound together by their attractive field-energies is denoted by $c^\pi$. A '*rigid',* electrically neutral, macroscopic body of crystalline-bound $c^\pi$ particles is then denoted by $C^\pi$. Unless stated otherwise, remarks in this paper concerning the *rotational propagation physics* of $c^\pi$ particles will apply equally well to $C^\pi$ bodies, and visa versa.

A very general kind of $C^\pi$ body used throughout is a spacecraft, denoted by $\underline{\Sigma}$, which houses a general scientific laboratory; including measurement standards for defining a Cartesian 3−distance ($\underline{x}^\alpha$) and time ($\underline{x}^0$ = c$\underline{t}$) relative to its origin ($\underline{O}$), plus, an inertial navigation system for controlling the orientation and propagation path of $\underline{\Sigma}$ to throughout the effective $^0\in_m$–continuum that remains after the calculated effects of matter gravity have been subtracted out by $\underline{\Sigma}$'s inertial navigation computer. Immediately following the derivation of GFT1, it will be shown—as a practical introduction to GFT2—that *all* inertial sensors (which necessarily includes *all* matter-gravity sensors)—provide unambiguous *proper* (in situ) measures of various *Maxwellian gravity fields* stemming from nothing other than temporal and spatial variations of the relative $\underline{\mathbf{p}}_m(\underline{x}^i)$ field.

## 2. The fundamental ${}^0\in_m$-reference frame, $\Sigma(x^i)$.

In Fig.1, $\Sigma(x^i)$ symbolizes the Cartesian measurement system of a copy of $\underline{\Sigma}$ which employs (1 + 3) coordinates $x^i = \{(x^0 = ct), x^\alpha\}$ relative to its origin O, but which is defined—for purely heuristic purposes—to be rotationally and rectilinearly at rest in the ${}^0\in_m$-continuum characterized by constant local values of $\in_m(x^i) = {}^0\in_m$ and $c(x^i) = c$.

### *a. The local $\Sigma^\ell$ and distant $\Sigma^d$ domains of $\Sigma$.*

In order to preserve the *fundamental status* and *symmetry* of $\Sigma$, while allowing its $x^i$ to parameterize the interactions of $\delta\mathcal{E}_m$ particles with long range $\delta\mathcal{E}_m$ fields like gravity, electricity, and magnetism—that would break $\Sigma$'s symmetry—it is necessary to divide the geometric domain of $\Sigma$ into a *local-laboratory* domain, $\Sigma^\ell$, *where all measurements are carried out*, and an abstract *distant-field* domain, $\Sigma^d$, in accordance with the following prescription:

The $\Sigma^\ell$ domain of $\Sigma$ remains free of any long range fields from distance sources, and contains the instruments and means for analyzing *local* $\delta\mathcal{E}_m$ particle interactions, and, any massive or massless *radiations* which might be emitted from *distant* $\delta\mathcal{E}_m$ particle interactions taking place within (or relative to) a given long-range field in the *distant-field* domain of $\Sigma$, $\Sigma^d$. As illustrated in Fig. 1, any reference to a long range field, such as $\mathbf{u}(x^i)$, presumes that $\mathbf{u}(x^i)$ is restricted to $\Sigma^d$ and that $\mathbf{u}(x^i) = \mathbf{0}$ within $\Sigma^\ell$.

It will be demonstrated herein, more definitively than previously[4], that $\Sigma^\ell$ is the reference frame in which *virtually* all of our present classical and quantum laws hold good—with the greatest possible physical significance.

## 3. The rectilinear and angular forms of $\delta\mathcal{E}_m$ propagation.

Let $\mathbf{w}(p) = w^\alpha\mathbf{e}_\alpha$ be the lightspeed-normalized, *rectilinear* propagation velocity of a given $p(x^i)$ relative to $\Sigma$.[a] And let $\varpi(c^\pi) = \varpi^\alpha\mathbf{e}_\alpha$ be the lightspeed normalized, *angular* propagation velocity of a given $c^\pi$ particle. Unless stated otherwise, $\mathbf{w} = \mathbf{w}(p^\pi \text{ or } C^\pi)$, and $\mathbf{w}^0 \equiv \mathbf{w}(p^0)$.

The ${}^0\mathcal{E}_m$ *rest status* of $\Sigma$ is then quantified by the statement, $\mathbf{w}(\Sigma) = \varpi(\Sigma) = \mathbf{0}$. And the essence of ${}^0\mathcal{E}$-Implication (iii) is then quantified by stating that the ${}^0\in_m$-continuum is characterized by a *lightspeed three scalar,* $s^0 \equiv [1 - (\mathbf{w}^0)^2] = 0$. Hence, $s^0 = 0$ is a *dynamical characterization* of the ${}^0\in_m$–continuum which holds good even if $c(x^i) < c$ where $\in_m(x^i) < {}^0\in_m$, provided that the *lightspeed normalization* of $\mathbf{w}$ is done with $c(x^i)$.

## 4. An arbitrary $\in_m$-reference frame, $\underline{\Sigma}(\underline{x}^i)$.

In Fig. 1, $\underline{\Sigma}(\underline{x}^i)$ denotes the more general (1 + 3) reference system of a spacecraft whose Cartesian space axes are co-aligned with those of $\Sigma$. The propagational state of $\underline{\Sigma}(\underline{x}^i)$ and the physical environment of $\underline{\Sigma}(\underline{x}^i)$ are thus completely arbitrary, and a double underscore will be used to label all variables associated with the origin $\underline{O}$ of $\underline{\Sigma}$. Some examples of which are $\underline{\underline{x}}^i = (\underline{\underline{\mathbf{r}}}, \underline{\underline{x}}^0)$ and $\underline{\underline{\beta}} = d\underline{\underline{\mathbf{r}}}/dx^0 = \underline{\underline{\mathbf{w}}} + \underline{\underline{\mathbf{u}}}$, with $\underline{\mathbf{r}} = \mathbf{0}$ defining $x^0 - \underline{x}^0 = 0$, and $\underline{\underline{\varpi}}(\underline{\Sigma})$ defining $\varpi(\underline{\Sigma})$ about the point $\underline{O}$ at $\underline{\underline{x}}^i$. The generality of $\underline{\Sigma}$ is then stated more precisely by saying that $\underline{\underline{\mathbf{w}}}(\underline{\Sigma})$ and $\underline{\underline{\varpi}}(\underline{\Sigma})$ are both generally unrestricted, and, $\underline{\Sigma} = \underline{\Sigma}^d = \underline{\Sigma}^\ell$ can be embedded in an arbitrary $\mathbf{p}_m(x^i)$ field of distant matter.

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[a] All rectilinear and angular velocities are normalized to lightspeed, unless stated otherwise.

The generality of $\underline{\Sigma}$ will not be exploited until after until GFT1 is derived, however—prior to which $\underline{\Sigma}$ will be restricted to an *inertial* $\underline{\Sigma}$ defined by $\underline{\underline{\varpi}}(\underline{\Sigma}) = \mathbf{0}$, and $\underline{\underline{\beta}} = \underline{\underline{\mathbf{w}}}(\underline{\Sigma})$ = constant in the $\underline{\underline{\beta}}$–coupling domain of the 3−space-energy continuum defined above.

*a. On the expected* **w**–*deformations of* $\underline{\Sigma}$*'s structure and properties.*

Given that $\underline{\Sigma}$'s structure is sustained by the relatively tenuous electromagnetic interactions of its relatively insubstantial $\delta\mathcal{E}_m$ particles, one can reasonably expect to find that the structure and properties of $\underline{\Sigma}$ are altered by it $\underline{\underline{\mathbf{w}}}$ in well defined ways which become increasingly discernable with respect to $\Sigma$ as $\underline{\underline{\mathbf{w}}}^2 \to 1$. And **w**-gauge theory nicely precipitates the precise forms of five such **w**−deformations (the five $\underline{\underline{\mathrm{w}}}\mathrm{d}^i$ of **w**-gauge theory) having a useful *two-fold degeneracy* which stems from, what can be considered to be, the limiting $\underline{\underline{\beta}}$ and $\underline{\mathbf{u}}$ causes of $\underline{\underline{\mathbf{w}}} = (\underline{\underline{\beta}} - \underline{\mathbf{u}})$. An invaluable tool for the derivation and understanding of the $\underline{\underline{\mathrm{w}}}\mathrm{d}^i$—the abstraction of a propagationally un-deformable reference frame—is therefore described next.

5. The working abstraction of a propagationally un-deformable reference frame, $`\Sigma(`x^i)$.

I define $`\Sigma = `\Sigma(`x^i)$ as a generic working abstraction of $\Sigma$ which acquires the label $`\underline{\Sigma} = `\underline{\Sigma}(`\underline{x}^i)$ when it is abstractly overlaid on $\underline{\Sigma}$; so as to be both co–aligned with and co-moving with $\underline{\Sigma}$. The abstractly un-deformable measurement standards and measures of $`\underline{\Sigma}$ are therefore immune to any deformations that the instruments and measures of $\underline{\Sigma}$ might suffer in response to the presence of a constant $\mathbf{p}_m$–field in $`\underline{\Sigma}$, $`\underline{\mathbf{p}}_m(`\underline{x}^i)$. The (1 + 3) coordinates of $\Sigma$ and $`\underline{\Sigma}$ are thus usefully related by an ordinary Galilean transformation; $\mathbf{r} = (\underline{\mathbf{r}} + `\underline{\mathbf{r}}) = (\underline{\underline{\beta}}x^0 + `\underline{\mathbf{r}})$, and $x^0 = `\underline{x}^0$.

We will also be overlaying the origin $`O$ of $`\Sigma$ on $\in_m(x^i)$ to obtain $`\Sigma_\in = `\Sigma_\in(`x_\in^i)$ as a working abstraction of an un-deformable $\Sigma$ reference frame that co-moves with the energy density $\in_m(x^i)$—in a *null propagation state* quantified, with respect to $\Sigma$, by $\mathbf{w}(`\Sigma_\in) = \varpi(`\Sigma_\in) = \mathbf{0}$. Relative to $\Sigma$ then, $`\Sigma_\in$ has a rectilinear velocity $\beta_\in = \mathbf{u}$, and (as explained in II.D.8) a spin-rate $\omega(`\Sigma_\in) = \mathbf{s}(`\Sigma_\in) = ½(\nabla\times\mathbf{u}(x^i))$.

Via these two applications, $`\Sigma = `\Sigma(`x^i)$ will be seen to provide an invaluable tool for the derivation and understanding of **w**-gauge theory.

*a. Useful formulas consistent with the Galilean transformation linking* $\Sigma$ *and* $`\underline{\Sigma}$.

Given:

$$d\mathbf{r} = (d\underline{\mathbf{r}} + d`\underline{\mathbf{r}}) = (\underline{\underline{\beta}}dx^0 + d`\underline{\mathbf{r}}),\ \text{ and }\ dx^0 = d`\underline{x}^0, \tag{2.4a}$$

it follows that:

$$\beta = \underline{\underline{\beta}} + `\underline{\beta}\,, \tag{2.4b}$$

and

$$dx^0 = \frac{dr}{\beta} = \frac{d`\underline{r}}{`\underline{\beta}} = d`\underline{x}^0. \tag{2.4c}$$

This shows that the form of the equation defining a $\delta\mathcal{E}_m$ particle's GRV is invariant under a Galilean transformation of the (1 + 3) coordinates linking $\Sigma$ and $`\underline{\Sigma}$, which naturally conserves the respective, GRS-defined, transport-time intervals. These *Galilean coordinate relations,* as they will be hereafter referred to, will be liberally employed throughout Section III to transform the intuitively derived $`\underline{\Sigma}$–$\underline{\Sigma}$ expressions of the $\underline{\underline{\mathrm{w}}}\mathrm{d}^i$ formulas into their experimentally verifiable $\Sigma$–$\underline{\Sigma}$ expressions, thereby establishing $`\underline{\Sigma}$'s utility as a very practical analytic perspective of $\delta\mathcal{E}_m$−$\in_m$ coupling.

6. Four principal analysis points and parameters.

For the derivations which follow, $x^i$, $\underline{x}^i$, and $`\underline{x}^i$ will generally parameterize the same arbitrary point-property of a $\delta\mathcal{E}_m$ field-particle field-point, $p(x^i)$, $\underline{p}(\underline{x}^i)$, and $`\underline{p}(`\underline{x}^i)$, from the complementary perspectives of $\Sigma$, $\underline{\Sigma}$, and $`\underline{\Sigma}$. Whereas $(\underline{\underline{x}}^i)$ will point exclusively to the origin $\underline{O}$ of $\underline{\Sigma}$ and thus to the origin $`\underline{O}$ of $`\underline{\Sigma}$ . As detailed below, a defined function of a given variable will then be similarly underscored to denote which of these reference systems its variable belongs to.

*a. The form invariance of operator, vector, scalar, and tensor functions.*

Let $\mathfrak{R}$, $\mathbf{w}$, f, and $T_{ik}$, denote a given operator, vector, scalar, and tensor, associated with $p(x^i)$ of $\Sigma^d$.

Then, $\underline{\mathfrak{R}}$, $\underline{\mathbf{w}}$, $\underline{f}$, and $\underline{T}_{ik}$ will denote the same mathematical quantities associated with $\underline{p}(\underline{x}^i)$ of $\underline{\Sigma}$.

And, $`\underline{\mathfrak{R}}$, $`\underline{\mathbf{w}}$, $`\underline{f}$, and $`\underline{T}_{ik}$ will denote the same mathematical quantities associated with $`\underline{p}(`\underline{x}^i)$ of $`\underline{\Sigma}$.

And, $\underline{\underline{\mathfrak{R}}}$, $\underline{\underline{\mathbf{w}}}$, $\underline{\underline{f}}$, and $\underline{\underline{T}}_{ik}$ denote the same mathematical quantities associated with $\underline{O}$ of $\underline{\Sigma}$ and $`\underline{O}$ of $`\underline{\Sigma}$.

Thus if the propagation velocity of a given $p^\pi$ particle is $\mathbf{w}$, $\underline{\mathbf{w}}$, and $`\underline{\mathbf{w}}$ referenced to $\Sigma$, $\underline{\Sigma}$, and $`\underline{\Sigma}$, and the particle's coupling strength referenced to $\Sigma$ is defined by $\ell = \ell(w) = 1/[1 - \mathbf{w}^2]^{½}$, we can subsequently introduce $\underline{\ell}$ and $`\underline{\ell}$ with the understanding that $\underline{\ell}$ and $`\underline{\ell}$ are the same functions of $\underline{\mathbf{w}}$ and $`\underline{\mathbf{w}}$ that $\ell$ is of $\mathbf{w}$, and that they describe the $p^\pi$ particle's propagational coupling strength referenced, respectively, to $\underline{\Sigma}$ and $`\underline{\Sigma}$.

7. A **D**(P/R) notation for clarifying *generalized ($\boldsymbol{p}_m$–field* + $\delta\mathcal{E}_m$*-field-particle)* equations.

The GRV equation, $\beta = (\mathbf{u} + \mathbf{w})$, is a primary example of what is referred to herein as a *generalized($\boldsymbol{p}_m$–field* + $\delta\mathcal{E}_m$ *field-particle*) equation, more concisely referred to as a *generalize field-particle* (GFP) equation, since it is an equation that combines *one type* of $\mathbf{p}_m$–field and *one type* of $\delta\mathcal{E}_m$–particle property—in this example the normalized $\mathbf{p}_m$–field; $\mathbf{p}_m/\epsilon_m = \mathbf{u}$, and the $\delta\mathcal{E}_m$ propagation velocity $\mathbf{w}$. Two other GFP's; the *generalized angular velocity* (GAV) equation, $\omega = \mathbf{s} + \varpi$, and the *generalized rectilinear acceleration* (GRA) equation, $\beta_0 = \mathbf{u}_0 + \mathbf{w}_0$, are derived in II.D.8,10.

The physics and the symmetries involved in each GFP can be conceptually and mathematically clarified by employing a **D**(P/R) qualification for each 3-vector, **D,** in a GFP equation, with the understanding that **D**(P/R) means the dynamic state **D** (of a specified $\delta\mathcal{E}_m$ particle P/relative to the frame of reference R that has been uniquely associated with **D**).

Having already described each of the parameters in the GRV, we may then strive to clarify its meaning by expressing the GRV in its **D**(P/R) form

$$\beta(p(x^i)/\Sigma^d) = \mathbf{u}(\epsilon_m(x^i)/\Sigma^d) + \mathbf{w}(p(x^i)/`\Sigma_\epsilon). \tag{2.3b}$$

However, since all terms in a GFP must pertain to the same $x^i$, we can always use the neater **D**(P/R) form

$$\beta(p/\Sigma^d) = \mathbf{u}(\epsilon_m/\Sigma^d) + \mathbf{w}(p/`\Sigma_\epsilon). \tag{2.3c}$$

This betrays, most clearly, what might at first glance seem to be a troublesome fact: Namely, that different terms in a GFP equation pertain to different reference systems. However, (2.3c) is a logical and practical equation of continuum mechanics which has naturally surfaced here as the founding equation of hyper-energy theory, and which (as mentioned in II.C.2) is employed in the self consistent form

$$\mathbf{w}(p/`\Sigma_\in) = \beta(p/\Sigma^d) - \mathbf{u}(\in_m/\Sigma^d), \tag{2.3d}$$

to formally source the tensor form and substance of Einstein-Maxwell gravity (as mentioned in II.C.2.a).

Hence, with the exception of $\mathbf{D} = \mathbf{w}$ (which is always referenced to $`\Sigma_\in$), the reference frame that $\mathbf{D}$(P) is both referred to and defined in is uniquely betrayed by the specific labeling assigned to $\mathbf{D}$. And with the exception of $\mathbf{w}$, it is understood that 3-vectors which are respectively labeled as ($\mathbf{D}$ or $\underline{\underline{\mathbf{D}}}$), $`\underline{\mathbf{D}}$, and $\underline{\mathbf{D}}$, are always respectively referenced to ($\Sigma^d$), $`\underline{\Sigma}$, and $\underline{\Sigma}$. This offers some justification for working with the original GRV form

$$\beta(p) = \mathbf{u}(\in_m) + \mathbf{w}(p). \tag{2.3a}$$

Furthermore, simply acknowledging that all terms—other than the *field* term—are *particle* parameters, offers some justification for working with the simplest form of all,

$$\beta = \mathbf{u} + \mathbf{w}. \tag{2.3d}$$

8. A first order Angular Complement to the GRV equation.

*a. The generalized angular velocity(GAV).*

Just as $\beta = (\mathbf{u} + \mathbf{w})$ is the generalized *rectilinear* velocity, referenced to $\Sigma^d$, of a $\delta\mathcal{E}_m$ particle (p) which has a *rectilinear* propagation velocity, $\mathbf{w}$, at $x^i$ of $\Sigma^d$ where the $\in_m$-continuum is *flowing rectilinearly* with velocity $\mathbf{u}$,

$$\omega(c^\pi) = \boldsymbol{s}(\in_m) + \varpi(c^\pi), \tag{2.5a}$$

is the generalized *angular* velocity (GAV), referenced to $\Sigma^d$, of a $c^\pi$ particle which has an *angular* propagation velocity, $\varpi$, at $x^i$ of $\Sigma^d$ where the $\in_m$-continuum is *circulating* with a vorticity $\boldsymbol{\zeta}(x^i) = \nabla\times\mathbf{u} \equiv 2\boldsymbol{s}$ which is presumed to be constant over the 3-volume of the $c^\pi$ particle. The parameter, $\boldsymbol{s}$, is the $c^\pi$–*spin equivalence* of $\in_m$–vorticity, $\boldsymbol{\zeta}$, since it accounts fully for $\omega(c^\pi)$ when $\varpi(c^\pi) = 0$.

In $\mathbf{D}$(P/R) form

$$\omega(c^\pi/\Sigma^d) = \boldsymbol{s}(\in_m/\Sigma^d) + \varpi(c^\pi/`\Sigma_\in). \tag{2.5b}$$

*b. Derivation of the GAV equation for $\underline{\Sigma}$.*

We now employ $\underline{\Sigma}$, the abstraction $`\underline{\Sigma}$, and the crucial fact that $\mathbf{D}(`\underline{\Sigma}/\Sigma^d) = \mathbf{D}(\underline{\Sigma}/\Sigma^d)$, to derive (2.5)—in a way that neatly circumvents the issues associated with; 1) the un-observability in $\underline{\Sigma}$ of $`\underline{\mathbf{u}}$ (derived in Section III), and 2) the complications of both $\mathbf{w}$-deformations and $\varpi$–deformations in $\underline{\Sigma}$.

Relative to $\Sigma^d$; $\mathbf{r} = \underline{\mathbf{r}}(\underline{O}) + `\underline{\mathbf{r}}(\underline{\Sigma})$, the dynamic *coordinate state* of $\underline{\Sigma}$ is parameterized by $\underline{\underline{\beta}}(\underline{O})$ and $\underline{\underline{\omega}}(\underline{\Sigma})$, and the dynamic *propagational state* of $\underline{\Sigma}$ is parameterized by $\underline{\underline{\mathbf{w}}}$ and $\underline{\underline{\varpi}}$. Referenced to $`\underline{\Sigma}$ then, $\in_m(`\underline{\mathbf{r}})$ is flowing with velocity $`\underline{\mathbf{u}}$ relative to $`\underline{\mathbf{r}}$. And because of $`\underline{\Sigma}$'s $\underline{\underline{\beta}}$ and $\underline{\underline{\omega}}$ relative to $\Sigma^d$, $\in_m(`\underline{\mathbf{r}}) = \in_m(\mathbf{r})$ must then be flowing relative to $\mathbf{r}$ of $\Sigma^d$ with velocity

$$\mathbf{u}(\mathbf{r}) = `\underline{\mathbf{u}}(`\underline{\mathbf{r}}) + \underline{\underline{\beta}}(`\underline{\Sigma}) + \underline{\underline{\omega}}(`\underline{\Sigma})\times`\underline{\mathbf{r}}. \tag{2.6}$$

And since $\underline{\underline{\beta}}$ and $\underline{\underline{\omega}}$ are coordinate point-functions (unaffected by the $`\nabla$ operator), taking the curl of (2.6) generates the following fundamental relationship between $`\boldsymbol{\zeta}$, $\boldsymbol{\zeta}$, and $\underline{\underline{\omega}}$:

$$\boldsymbol{\zeta}(x^i) \equiv \nabla\times\mathbf{u} = `\underline{\boldsymbol{\zeta}} + \underline{\underline{\omega}}(`\nabla\bullet`\underline{\mathbf{r}}) - (\underline{\underline{\omega}}\bullet`\nabla)`\underline{\mathbf{r}}, \tag{2.7a}$$

$$= `\underline{\boldsymbol{\zeta}} + 2\underline{\underline{\omega}}.^5 \tag{2.7b}$$

In $\mathbf{D}$(P/R) form

$$\boldsymbol{\zeta}(\in_m/\Sigma^d) = `\boldsymbol{\zeta}(\in_m/`\underline{\Sigma}) + 2\underline{\underline{\omega}}(`\underline{\Sigma}/\Sigma^d). \tag{2.7c}$$

We then recall the independent fluid-dynamic formula

$$½`\boldsymbol{\zeta}\,(\in_m/`\underline{\Sigma}) = -\,\underline{\underline{\varpi}}(`\underline{\Sigma}/`\Sigma_\in), \tag{2.8}$$

which explains, in this case, that one half of the $\in_m$-vorticity relative to $`\underline{\Sigma}$ (at the point $`\underline{O}$) is, necessarily, always opposite and equal to the angular propagation rate of $`\underline{\Sigma}$ relative to $`\Sigma_\in$. This is derived here for the special case that $\underline{\underline{\varpi}} = \underline{\underline{\varpi}}^z\,\mathbf{e_z}$. This allows us to employ cylindrical coordinates ($`\underline{r}$,$`\underline{\theta}$,$`\underline{z}$) to state that, since $\underline{\underline{\varpi}}^z(`\underline{\Sigma}/`\Sigma_\in)$ is relative to $`\Sigma_\in$, then, regardless of how $\in_m$ may or may not be circulating relative to $\Sigma^d$, we are assured that, relative to $`\underline{\Sigma}$

$$`\underline{\mathbf{u}}_\theta = -\,(\underline{\underline{\varpi}}^z)`\underline{\mathbf{r}}. \tag{2.9}$$

By employing Stokes' theorem and $d`\underline{\mathbf{r}} = `\underline{\mathbf{r}}\; d`\underline{\theta}$, we then obtain

$$\oint `\underline{\mathbf{u}}\bullet d`\underline{\mathbf{r}} = -\,2\pi`r^2\underline{\underline{\varpi}}^z = -\,2`\mathbf{A}\bullet\underline{\underline{\varpi}}^z, \tag{2.10}$$

$$= \oiint `\boldsymbol{\zeta}\bullet d`\mathbf{A} = +\,`\mathbf{A}\bullet`\boldsymbol{\zeta}^z. \tag{2.11}$$

Thus proving that

$$½`\boldsymbol{\zeta}^z = -\,\underline{\underline{\varpi}}^z, \tag{2.8}$$

is a direct consequence of the 100% coupling of $\underline{\Sigma}$ and the $\in_m$-continuum. Finally, recalling that $\mathbf{D}(`\underline{\Sigma}/\Sigma^d) = \mathbf{D}(\underline{\Sigma}/\Sigma^d)$, we let $\mathbf{s} \equiv ½\boldsymbol{\zeta}$ define the $c^\pi$-*spin equivalence* of $\boldsymbol{\zeta}$, and insert (2.8) into (2.7b) to obtain the GAV of $\underline{\Sigma}$ in the two equivalent forms:

$$\underline{\underline{\omega}}(\underline{\Sigma}/\Sigma^d\,) = \boldsymbol{s}(\in_m/\Sigma^d) + \underline{\underline{\varpi}}(\underline{\Sigma}/`\Sigma_\in), \tag{2.12}$$

and

$$\omega(c^\pi/\Sigma^d) = \boldsymbol{s}(\in_m/\Sigma^d) + \varpi(c^\pi/`\Sigma_\in), \tag{2.5b}$$

where $\mathbf{s}$ is now presumed to be constant over the 3-volumes of $\underline{\Sigma}$ and the $c^\pi$ particle.

The GRV formula introduced a previously unknown *dual physical basis* for the observable rectilinear velocity of a p particle in a general (*non inertial*) region of the 3-space-energy continuum characterized by $\mathbf{u}(x^i) \neq \mathbf{0}$, and $\in_m \leq {}^0\in_m$. Namely: *Frictionless rectilinear propagation* of a p particle relative to its underlying $\in_m$-continuum, and, the *local rectilinear convection* of $\in_m$ relative to the superficially undisturbed ${}^0\in_m$-continuum.

Now, the complementary GAV has introduced a previously unknown *dual physical basis* for the observable angular velocity of a $c^\pi$ particle in a general (*non inertial*) region of the 3-space-energy continuum characterized by $\mathbf{s}(x^i) \neq \mathbf{0}$, and $\in_m \leq {}^0\in_m$. Namely: The *frictionless angular propagation* of a $c^\pi$ particle relative to its underlying $\in_m$-continuum, and, the *local vorticity* of $\in_m$ relative to the superficially undisturbed ${}^0\in_m$-continuum.

9. On the intrinsic unification of *polar* and *axial* $\mathbf{p}_m(x^i)$-field and particle properties in the (1 + 3) sector of hyper-energy theory.

The GRV equation puts two distinct field-theoretic concepts on the same mathematical footing: The distinct *rectilinear* field-particle-property, $\mathbf{W}$(p), and the distinct *polar* field-property, $\mathbf{u} = \mathbf{p}_m(x^i)/\in_m$.

Whereas the GAV equation puts two other distinct theoretical concepts on the same mathematical footing: The distinct *angular* field-particle-property, $\omega(c^p)$, and the distinct *axial* field-property, $\underline{\boldsymbol{\zeta}}(\in_m)$, together with its ½ spin equivalency for $c^\pi$ particles and, possibly, certain 'point-like' particles as well.

10. A second order Rectilinear-Acceleration complement to the GRV equation.

*a. The generalized rectilinear acceleration (GRA).*

Applying the $(\partial/\partial x^0)$ operator to the GRV equation, and defining:

$$\beta_0 = \partial\beta/\partial x^0 = \beta_0(p/\Sigma^d), \tag{2.13a}$$

$$\boldsymbol{u}_0 = \partial\mathbf{u}/\partial x^0 = \mathbf{u}_0(\in_m/\Sigma^d), \tag{2.13b}$$

$$\mathbf{w}_0 = \partial\mathbf{w}/\partial x^0 = \mathbf{w}_0(p/`\Sigma_\in), \tag{2.13c}$$

yields the *generalized rectilinear acceleration* (GRA) equation,

$$\beta_0 = \mathbf{u}_0 + \mathbf{w}_0 \tag{2.14a}$$

of an arbitrary p particle at $x^i$ of $\Sigma^d$, stating that

$$\beta_0(p/\Sigma^d) = \mathbf{u}_0(\in_m/\Sigma^d) + \mathbf{w}_0(p/`\underline{\Sigma}_\in). \tag{2.14b}$$

11. Can the *relative manifestation* $`\underline{\mathbf{F}}$ of the *absolute* $\in_m$-field $\mathbf{F}$ in a GFP equation be entirely independent of $\mathbf{F}$, and due entirely to the *particular propagational state* of $\underline{\Sigma}$ defined by the GFP equation?

The answer, yes, which is gainfully employed in Section III, is explained for the $`\underline{\mathbf{u}}$, $`\underline{\mathbf{s}}$, and $`\underline{\mathbf{u}}_0$ fields of the GRV, GAV, and GRA equations by creating *relative forms* of the GFP's as follows:

*a. The $`\underline{\mathbf{u}}$ field of the relative GRV equation.*

Recalling that $\mathbf{D}(`\underline{\Sigma}/\Sigma^d) = \mathbf{D}((`\underline{\mathbf{r}})/\Sigma^d) = \mathbf{D}(\underline{\Sigma}/\Sigma^d)$, and rearranging the founding GRV equation to define its *relative form*

$$`\underline{\mathbf{u}}(\in_m(`\underline{\mathbf{r}})/`\underline{\Sigma}) \equiv [\mathbf{u}(\in_m(`\underline{\mathbf{r}})/\Sigma^d) - \underline{\beta}(\underline{\Sigma}/\Sigma^d)] = -\underline{\mathbf{w}}(\underline{\Sigma}/`\Sigma_\in), \tag{2.15}$$

reveals the following four things about the relative $\in_m$ velocity field ($`\underline{\mathbf{u}}$) that can exist in $`\underline{\Sigma}$:

- Via the equality of $`\underline{\mathbf{u}}$ and the bracketed middle term of (2.15), $`\underline{\mathbf{u}}$ is mathematically explained, from the perspective of $\Sigma^d$, as the flow velocity of $\in_m$ *relative to* $\underline{\Sigma}$—which is naturally biased by $\underline{\Sigma}$'s GRV relative to $\Sigma^d$.
- Via the equality of $`\underline{\mathbf{u}}$ and third term of (2.15), $`\underline{\mathbf{u}}$ is mathematically explained as the negative of $\underline{\Sigma}$'s propagation velocity relative to $`\Sigma_\in$.
- The $`\underline{\mathbf{u}}$ field of the relative GRV equation is, therefore, *totally independent* of the flow velocity of the $\in_m$-continuum, being *entirely dependent*, instead, on the velocity of $\underline{\Sigma}$'s rectilinear propagation relative to $`\Sigma_\in$.
- If $\underline{\beta}(\underline{\Sigma}/\Sigma^d) = \mathbf{0}$, then $`\underline{\mathbf{u}}(\in_m(`\underline{\mathbf{r}})/`\underline{\Sigma})$ is identical with $\mathbf{u}(\in_m(`\underline{\mathbf{r}})/\Sigma^d)$, as one would expect.

*b. The $`\underline{\mathbf{s}}$ field of the relative GAV equation.*

Rewriting the GAV equation to define

$$`\underline{\mathbf{s}}(\in_m(\underline{O})/`\Sigma) \equiv [\mathbf{s}(\in_m(\underline{O})/\Sigma^d) - \underline{\omega}(\underline{\Sigma}/\Sigma^d)] = -\underline{\varpi}(\underline{\Sigma}/`\Sigma_\in), \tag{2.16}$$

reveals the following three similar things about the ½-vorticity field ($\grave{\underline{\boldsymbol{s}}}$) that can exist in $\grave{\underline{\Sigma}}$:

- Via the equality of $\grave{\underline{\boldsymbol{s}}}$ and the second term (middle) of (2.16), $\grave{\underline{\boldsymbol{s}}}$ is mathematically explained, from the perspective of $\Sigma^d$, as the ½–vorticity of $\in_m$ *relative to* $\underline{\Sigma}$—which is naturally biased by $\underline{\Sigma}$'s GAV relative to $\Sigma^d$.
- Via the equality of $\grave{\underline{\boldsymbol{s}}}$ and third term of (2.16), $\grave{\underline{\boldsymbol{s}}}$ is mathematically explained as the negative of $\underline{\Sigma}$'s angular propagation velocity relative to $\grave{\Sigma}_\in$.
- The $\grave{\underline{\boldsymbol{s}}}$ field of the relative GAV equation is, therefore, *totally independent* of the vorticity of the $\in_m$-continuum, being *entirely dependent*, instead, on the velocity of $\underline{\Sigma}$'s angular propagation relative to $\grave{\Sigma}_\in$.
- If $\underline{\underline{\omega}}(\underline{\Sigma}/\Sigma^d) = \mathbf{0}$, then $\grave{\underline{\boldsymbol{s}}}(\in_m(\underline{O})/\grave{\underline{\Sigma}})$ is identical with $\boldsymbol{s}(\in_m(\underline{O})/\Sigma^d)$, as one would expect.

*c. The $\grave{\underline{\mathbf{u}}}_0$ field of the relative GRA equation.*

Likewise, rearranging the GRA equation to define the relative GRA equation

$$\grave{\underline{\mathbf{u}}}_0(\in_m(\grave{\underline{\mathbf{r}}})/\grave{\underline{\Sigma}}) = [\mathbf{u}_0(\in_m(\grave{\underline{\mathbf{r}}})/\Sigma^d) - \underline{\underline{\beta}}_0(\underline{\Sigma}/\Sigma^d)] = -\underline{\underline{\mathbf{w}}}_0(\underline{\Sigma}/\grave{\Sigma}_\in), \tag{2.17}$$

reveals the following three things about the $\in_m$ acceleration field ($\grave{\underline{\mathbf{u}}}_0$) that can exist in $\grave{\underline{\Sigma}}$:

- Via the equality of $\grave{\underline{\mathbf{u}}}_0$ and the second (middle) term of (2.17), $\grave{\underline{\mathbf{u}}}_0$ is mathematically explained, from the perspective of $\Sigma^d$, as the explicit acceleration of $\in_m$ *relative to* $\underline{\Sigma}$—which is naturally biased by $\underline{\Sigma}$'s GRA relative to $\Sigma^d$.
- Via the equality of $\grave{\underline{\mathbf{u}}}_0$ and third term of (2.17), $\grave{\underline{\mathbf{u}}}_0$ is mathematically explained as the negative of $\underline{\Sigma}$'s propagational acceleration relative to $\grave{\Sigma}_\in$.
- The $\grave{\underline{\mathbf{u}}}_0$ field of the relative GAV equation is, therefore, *totally independent* of the explicit acceleration of the $\in_m$-continuum, being *entirely dependent*, instead, on the explicit acceleration of $\underline{\Sigma}$'s rectilinear propagation relative to $\grave{\Sigma}_\in$.
- If $\underline{\underline{\beta}}_0(\underline{\Sigma}/\Sigma^d) = \mathbf{0}$, then $\grave{\underline{\mathbf{u}}}_0(\in_m(\grave{\underline{\mathbf{r}}})/\grave{\underline{\Sigma}})$ is identical with $\mathbf{u}_0(\in_m(\grave{\underline{\mathbf{r}}})/\Sigma^d)$, as one would expect.

*d. On the intrinsic un-observability of a constant $\grave{\underline{\boldsymbol{u}}}$ field, and the intrinsic observability of the $\grave{\underline{\boldsymbol{s}}}$ and $\grave{\underline{\boldsymbol{u}}}_0$ fields, and thus, the field $\grave{\underline{\mathbf{g}}} = \grave{\underline{\nabla}}(\tfrac{1}{2}\grave{\underline{\boldsymbol{u}}}^2)$ covering Newtonian gravity.*

In Section III.B, we be begin the derivation of **w**-gauge theory by deducing that a $\grave{\underline{\mathbf{p}}}_m(\grave{\underline{x}}^i)$ field must be un-observable, and by raising this conclusion to the level of an axiom, called the Null-$\underline{\mathbf{p}}$ Axiom, which states that the non-zero physical state described by $\grave{\underline{\mathbf{p}}}_m(\grave{\underline{x}}^i) \neq \mathbf{0}$ in $\grave{\underline{\Sigma}}$ must be consistent with the null physical state described by $\underline{\mathbf{p}}_m(\underline{x}^i) = \mathbf{0}$ in $\underline{\Sigma}$. The Null-$\underline{\mathbf{p}}_m$ Axiom and GRV formula are then consistently employed to derive the full $\underline{\underline{w}}\mathrm{d}^i$ structure of **w**-gauge theory, which is shown to cover special relativity theory in the special ($\mathbf{u} = \mathbf{0}$) limit of the γ–coupling domain.

Then in Section III.N, as useful practical prelude to EM gravity, we delineate a 3-vector theory of Maxwellian gravity which allows us to derive experimentally confirmed scale factors for the *proper* (in situ) measurements of the $\in_m$–fields $\grave{\underline{\boldsymbol{s}}}$ and $\grave{\underline{\mathbf{u}}}_0$, which (measurements) are routinely made today with the so-called *inertial sensors* of the many and various inertial navigation systems that are in use today. This conveniently provides the opportunity to deduce two things via **w**-gauge theory that are readily confirmed by EM gravity. Namely: Maxwellian matter-gravity—covering Newtonian matter-gravity—is the $\grave{\underline{\mathbf{p}}}_m$ field, $\grave{\underline{\mathbf{g}}} = \grave{\underline{\nabla}}(\tfrac{1}{2}\grave{\underline{\mathbf{u}}}^2)$. And the singularity in general relativity theory corresponding to a gravitational black hole is the thermodynamic Mach-one condition in hyper-energy theory wherein the flow velocity of $\in_m$ equals the local velocity of light.

III. GFT1: A **w-**Gauge Theory of $\in_m$-Field-Particle Coupling.

A. Three distinct environments of $\in_m$-field-particle coupling which remove an intrinsic three-fold degeneracy of propagation velocity.

Given that **w** is a natural parameter for quantifying propagational deformations of $\delta\mathcal{E}_m$ particles and structures, and that the founding GRV formula give **w** = (β − **u**) with a three fold degeneracy; it is useful to precede the derivation of the ($\ell = 1/[1 - \mathbf{w}^2]^{1/2}$ proportional) $\underline{\underline{w}}d^i$ of w−gauge theory with brief descriptions of three distinct $\in_m$−particle coupling environments which remove the triple degeneracy of $\ell(\mathbf{w}^2)$ and which warrant unique notations for the propagational coupling strengths in each of those specific environments. The specific $\in_m$−particle coupling environments which remove the triple degen-eracy of **w** = (β − **u**) are

| Respectively characterized by: | With: | Defining a domain of: | With degenerate coupling strength: |
|---|---|---|---|
| **u** = **0**. Whence **w** = β. | $\in_m = {}^0\in_m$. | ${}^0\in_m$ γ-coupling | $\ell[(\mathbf{w} = \beta)^2] \equiv \gamma(\beta^2)$ |
| β = **0**. Whence **w** = − **u**. | $\in_m \leq {}^0\in_m$. | $\in_m$ Γ-coupling | $\ell[(\mathbf{w} = -\mathbf{u})^2] \equiv \Gamma(\mathbf{u}^2)$ |
| β = **u**. Whence **w** = **0**. | $\in_m \leq {}^0\in_m$. | $\in_m$ null-coupling | $\ell(0) = 1 \equiv$ Free Fall |

These special environments of $\in_m$−particle coupling are conveniently referred to hereafter as the degenerate (γ, Γ, and null)-coupling environments of w−gauge theory, with $\ell$−coupling representing the most general expression of the coupling. Some specific attributes and notations unique to each of these $\in_m$−particle coupling environments are describe as follows.

1. Analytic attributes and notations unique to the γ−coupling environment of **w-**gauge theory.

With the additional qualification that $s^0 = 0$, the γ-coupling environment of **w-**gauge theory covers what 20th century science referred to as *inertial space*, and it provides the most convenient coupling environment for deriving the $\underline{\underline{w}}d^i$ of **w-**gauge theory because it allows the ${}^0\in_m$−particle coupling to be described and thus viewed from the two complementary perspectives of

Σ: Where **w**(p) = β (p), and $\underline{\mathbf{w}}(\underline{\Sigma}) = \underline{\beta}$, parameterize, respectively, the coupling strengths $\ell$ = γ and $\underline{\ell} = \underline{\gamma}$ of an arbitrary $p^\pi$ particle and of $\underline{\Sigma}$ (or any $\delta\mathcal{E}_m$ structure that is either stationary in or co-propagating with $\underline{\Sigma}$).

`$\underline{\Sigma}$: Where the same $p^\pi$ particle is propagating in a constant `$\underline{\mathbf{p}}_m$ field, `$\underline{\mathbf{u}} = -\underline{\mathbf{w}}$, with rectilinear velocity, `$\underline{\beta}$ = `$\underline{\mathbf{w}}$ + `$\underline{\mathbf{u}}$, and a coupling strength `$\underline{\ell}$ that has only two degenerate values. Namely, `$\underline{\ell} \rightarrow$ `$\underline{\Gamma}$ for `$\underline{\beta} \rightarrow \mathbf{0}$, and `$\underline{\ell} \rightarrow 1$ for `$\underline{\mathbf{w}} \rightarrow \mathbf{0}$.

*a. The special Galilean invariance of w(p) and $\ell$(p).*

In the γ-coupling environment we have β = **w** = [$\underline{\beta}$ + `$\underline{\beta}$] = [$\underline{\mathbf{w}}$ + (`$\underline{\mathbf{w}}$ + `$\underline{\mathbf{u}}$)] = `$\underline{\mathbf{w}}$, and `$\underline{\ell} = \ell$. Hence, in a (u = 0)-environment both the *propagation velocity* and the *coupling strength* of a particle are invariant under a Galilean transformation of *un-deformed* (1 + 3) coordinates linking `$\underline{\Sigma}$ to Σ. Also, since `$\underline{\mathbf{u}} = -\underline{\beta} = -\underline{\mathbf{w}}$, it follows that `$\underline{\Gamma}$(`$\underline{\mathbf{u}}^2$) = $\underline{\ell}(\underline{\mathbf{w}}^2) = \underline{\gamma}(\underline{\beta}^2)$ are three equivalent expressions of the propagation coupling strength of $\underline{\Sigma}$ and any co-moving $\delta\mathcal{E}_m$ particles or structures. These special (u = 0) invariants and equalities constitute useful supplements the Galilean coordinate relations of (2.4).

2. Attributes and notations unique to the Γ−coupling environment of **w-**gauge theory.

Let a $p^{\pi}$ particle be at rest at $x^{\alpha}$ of $\Sigma^{d}$ and embedded in a $\mathbf{p}_{m}(x^{i})$ field which is broken out of ${}^{0}\in_{m}(x^{i})$ by the mass of a distant $\delta\mathcal{E}_{m}$ particle-structure, with $\mathbf{w}(x^{i}) = -\,\mathbf{u}(x^{i})$ accounting for the *rest status* of the $p^{\pi}$ particle in Σ. From the perspective of Σ then, the $p^{\pi}$ particle couples to the potential of Maxwellian gravity with a strength $\ell(\mathbf{w}^{2}) \equiv \Gamma(\mathbf{u}^{2})$, where $\mathbf{u}(x^{i}) = \mathbf{p}_{m}(x^{i})/\in_{m}(x^{i})$.

3. Attributes and notations unique to the null-coupling environment of **w-**gauge theory.

A $p^{\pi}$ particle which is embedded in the velocity field $\mathbf{u}(x^{i})$ of $\in_{m} \le {}^{0}\in_{m}$ at $x^{\alpha}$ of $\Sigma^{d}$ can be moving entirely with the flow of $\in_{m}$. A state which is characterized by $\beta = \mathbf{u}$ and thus by $\mathbf{w} = \mathbf{0}$. From the perspective of Σ then, the propagational coupling strength of a $p^{\pi}$ particle moving with the flow is unity because it is not propagating. It is simply being carried along with the flow of $\in_{m}$—in a state of propagational free fall.

B. On the un-observability of a constant $`\underline{\mathbf{p}}_{m}$ field via $\delta\mathcal{E}_{m}$ particle interactions in $\underline{\Sigma}$, and its gauge-field and symmetry implications.

From the perspective of Σ and Newtonian mechanics, $\underline{\Sigma}$'s total energy, $`\underline{E}$, is just the sum of its internal energy $`\underline{E}_{in}$ and its kinetic energy $`\underline{KE}$. The propagational state of $\underline{\Sigma}$ is then further quantified by the statement that $`\underline{E} = `\underline{E}_{in} + `\underline{KE}$. = constant. In addition, the measurement of a physical quantity (Q) requires that a certain amount of *Q-proportional measurement energy* be transferred to the Q-meter. Consequently, if a physical operation or process in $\underline{\Sigma}$ could produce a measure of $`\underline{\mathbf{p}}_{m}$, the $`\underline{\mathbf{p}}_{m}$−proportional increase of $`\underline{E}_{in}$ would have to be at the expense of a corresponding decrease of $\underline{\Sigma}$'s $`\underline{KE}$, in accordance with the formula:

$$d(`\underline{E}_{in}) = -\,d(`\underline{KE}) = -\,d(`\underline{\mathbf{P}}^{2})/2`\underline{M} = -\,`\underline{\mathbf{V}}\bullet d`\underline{\mathbf{P}}. \tag{3.1}$$

1. The Null-$\underline{\mathbf{p}}_{m}$ Axiom.

From the preceding considerations of first principles, it follows that the possibility of employing $\delta\mathcal{E}_{m}$ particle interactions in $\underline{\Sigma}$ to measure a constant $`\underline{\mathbf{p}}_{m}$ field must be excluded on the grounds that such a measurement would violate the well established conservation laws of $\delta\mathcal{E}_{m}$ energy and momentum. But Maxwell's gravity theorem guarantees that virtually every inertial system of reference is embedded in a constant $`\underline{\mathbf{p}}_{m}$ field. Thus, in order for Maxwell's gravity theorem to be preserved along with the conservation laws of $\delta\mathcal{E}_{m}$ energy and momentum, we are led to conclude that:

> The physical state described by a constant $`\underline{\mathbf{p}}_{m}\,(`\underline{x}^{i}) \neq \mathbf{0}$ in $`\underline{\Sigma}$ must be fully consistent with the null physical state in $\underline{\Sigma}$ described by $\underline{\mathbf{p}}_{m}(\underline{x}^{i}) = \mathbf{0}$.

I raise this conclusion to the level of an axiom, called the *Null-*$\underline{\mathbf{p}}_{m}$ *Axiom* (NPA), and note that the NPA has the following two corollaries:

NPA−Corollary One (NPA−C1)

> A $`\underline{\mathbf{p}}_{m}$ field is a fundamental gauge field whose various influences on the properties and physics of $\delta\mathcal{E}_{m}$ particle in $\underline{\Sigma}$ constitute new laws of $\delta\mathcal{E}_{m}$ particle physics which are valid in every inertial reference frame.

NPA−Corollary Two (NPA−C2)

> Insofar as the laws $\delta\mathcal{E}_{m}$ particles and fields are concerned, the physical space of every $\underline{\Sigma}$ is indistinguishable from the ${}^{0}\in_{m}$-continuum of Σ.

2. The inherent equivalence of the Null-$\underline{\mathbf{p}}_m$ Axiom and the two postulates of SRT.

The first postulate of SRT[6] can be interpreted as stating that:

> *The physical laws describing the interactions of (particles and fields) with (particles and fields) are the same in all inertial systems of reference.*

And the second postulate of SRT[7] can be interpreted as stating that:

> *With respect to any inertial reference system, electromagnetic radiation propagates with the same speed c irrespective of both the relative direction of the propagation and the relative velocity of the source of the radiation.*

Since NPA−C2 holds that the laws of $\delta\mathcal{E}_m$ particle physics can not distinguish the physical space of $\underline{\Sigma}^\ell$ from the $^0\in_m$-continuum of $\Sigma^\ell$, and $\underline{\Sigma}^\ell$ is an arbitrary inertial system of reference, it follows that the Null-$\underline{\mathbf{p}}_m$ Axiom covers the two postulates of SRT.

3. A useful consequence of NPA−C2.

NPA−C2 gives rise to the following working prediction of hyper-energy theory:

> Starting with the $\underline{\underline{w}}d^i$ of **w**-gauge theory, the *dynamical content* of every classical and quantum law of $\delta\mathcal{E}_m$ particle physics can be looked upon anew; as being referenced to the $^0\in_m$–continuum, in order to ascertain the new and collectively unifying physical significances that the laws of $\delta\mathcal{E}_m$ particle physics have for the formal structure of hyper-energy theory.

4. GFT1 as a pragmatic prerequisite to, but innate complement of, GFT2.

The derivation of GFT1 is driven by the explicit gauge-field character of the Null-$\underline{\mathbf{p}}_m$ Axiom stated by NPA−Corollary One. In Section IV, however, Einstein-Maxwell gravity is derived independently of **w-**gauge theory *per se* by employing

a) Eqs. (2.1−3),
b) The lightspeed 3-scalar, $s_3$, and,
c) The invariance of Maxwell's equations implied by NPA−C2,

to give the (1 + 3)−dimensional tensor calculus[a] a far greater reach and descriptive power—fully commensurate with Maxwell's gravity theorem. EM gravity will thus be seen to have an extraordinary efficacy due to the synergism of two factors:

- The revolutionary physical content of Eqs. (2.1−3).
- The much wider range of dynamics and symmetries which the expanded (1 + 3)−dimensional scalar source of EM gravity, $ds = dx^0/\ell$, makes available—covering $\delta\mathcal{E}_m$ particles and fields, and, the flow dynamics of the $\in_m$-continuum, and defining a fundamental tensor, $g_{ik}(u^\alpha)$, which represents a (1 + 3)-dimensional gauge-field prototype of Kaluza's (1 + 4)−dimensional gravitational-electromagnetic tensor.

It will thus become clear that EM gravity contains GFT1—but only implicitly. Hence the problem with by-passing GFT1 and proceeding directly to Section IV is that one would then be immediately

[a] That was precipitated from the static (0 + *p*)-dimensional tensor calculus by special relativity theory.

faced with no less than ten different physical ways that the coupling strength $\ell = 1/[1 - \mathbf{w}^2]^{½}$ is readily precipitated by the tensor mathematics (five for $\mathbf{w} \rightarrow \beta$ and five for $\mathbf{w} \rightarrow \mathbf{-u}$)—with no *a priori* understanding of the physical reasons for these different manifestations of $\ell$ or their relative ascendancy. The effort which would then have to be expended to unravel all of this, by effectively extracting **w-**gauge theory from EM gravity, would severely impede the mathematical exploitation and appreciation of EM gravity.

For these reasons **w-**gauge theory can be regarded as a very pragmatic prerequisite to, but inate complement of, EM gravity—that reveals that the actions of the $\grave{}\underline{\mathbf{p}}_m$ field which insure that $\underline{\mathbf{p}}_m(\underline{x}^i) = \mathbf{0}$, locally, are the actions of a vector potential of gravity that represents a broken symmetry of time—having scalar and vector components that are mathematically similar to the scalar and vector potentials of Maxwell's electrodynamics.

*a. The general Lorentz transformation that will be precipitated by both **w-**gauge theory and EM gravity.*

It will be shown in Sections III.(E–I) that the first three **w**−deformations of GFT1 provide a full and complete $\grave{}\underline{\mathbf{p}}_m$ gauge-field rationale for a general Lorentz transformation linking $dx^i$ and $d\underline{x}^i$. And in full support of the comments in Section III.B.2, the general Lorentz transformation will also be precipitated by EM gravity.

Consequently, it is important to know in advance that the general Lorentz transformation derived herein—initially from $\underline{\underline{w}}d^{1-3}$—is the general Lorentz transformation that is both indigenous to hyper-energy theory and in full accord with the purpose for which it was originally derived and applied via Poincaré and Lorentz.[8] A purpose which we now know was undermined simply and solely by the failure on the part of all 19th century theorists to perceive that the temporal component of the Lorentz transformations could be explained in the same causal ($\gamma$-proportional) way as the spatial component.[9] The fact that this failure is fully rectified by the w−gauge theory component of hyper-energy theory is, therefore, a large part of the reason why GFT1 is such a pragmatic prerequisite to, but inate complement of EM gravity.

*b. Historical note.*

The above noted failure on the part of all 19th theorists; to even venture the suspicion that propagating clocks might run slow by the $\gamma$ factor, accounts for the fact that no successful Lorentz invariant theory of the 3-space medium had been proposed prior to 1905. But some early relativity texts got this wrong; stating that such a theory had been proposed prior to 1905—implying, *unsatisfactorily so*. As Rindler apologetically noted: "I must hasten to include myself among the contributors to this myth,[10,11] and regret to have to include in the same category my revered friend Professor Peter Bergmann whose 1942 book[12] has educated many relativists."

Historical accidents of this kind have helped to establish a false belief that all possible alternatives to special relativity theory have been exhausted, and that special relativity theory is therefore wholly unique and unassailable.

C. The four generators of **w-**gauge theory.

The Null-$\underline{\mathbf{p}}_m$ Axiom and the GRV equation must be preserved by the laws of $\delta\mathcal{E}_m$ particle physics. In this sense the Null-$\underline{\mathbf{p}}_m$ Axiom and the GRV equation constitute two generators; (G1) and (G2), of a gauge-field structure of $\delta\mathcal{E}_m$ particle physics that will be consistent with Maxwell's gravity theorem. The need to also preserve Maxwell's *electrodynamic* field laws (G3) and Newton's *momentum conservation* laws (G4) defines two additional generators of the gauge-field structure of $\delta\mathcal{E}_m$ particle physics which are to be delineated by **w-**gauge theory.

Hence, G1−4 will now be employed—in the γ-coupling environment—to derive a set of five $\grave{\underline{\mathbf{p}}}_m$–particle coupling deformations ($\underline{\underline{w}}d^i$) which are slaved to a single coupling strength ($\underline{\underline{\ell}}$) and which therefore act *conjointly* to preserve G1−4 and to define **w-**gauge theory.

D. Outline of $\underline{\underline{w}}d^i$ derivations.

1. $\underline{\underline{w}}d^1$: The fundamental **w-**compression of 'rigid' $\delta\mathcal{E}_m$ 3-volumes.

The inhomogeneous wave equation of G3 is solved to obtain the soliton-like electromagnetic potentials ($\varphi$, $\mathbf{A}^* = c\mathbf{A}$) which physically envelop and co-propagate with a given electric charge situated at the origin of $\underline{\Sigma}$ and $\grave{\underline{\Sigma}}$. Such fields are shown to contain an asymmetry in the direction of $\grave{\underline{\mathbf{u}}}$ which would violate the Null-$\underline{\mathbf{p}}_m$ Axiom if it could be observed in $\underline{\Sigma}$. It is then shown that the $\grave{\underline{\mathbf{u}}}$–asymmetry will be rendered un-measurable in $\underline{\Sigma}$ if the otherwise 'rigid' 3−volume ($\grave{\underline{V}}$) of any $\delta\mathcal{E}_m$ structure is propagationally compressed along the direction of $\grave{\underline{\mathbf{u}}}$, by the factor $\grave{\underline{\Gamma}}^{-1}$. This $\underline{\underline{w}}d^1$ component of **w-**gauge theory is subsequently identified as the spatial component of a general Lorentz transformation—but it is pictorially defined for this outline by $\underline{\underline{w}}d^1 = V^0/V_{||} = \ell$, where $V^0$ is rest 3-volume of a give $\delta\mathcal{E}_m$ structure, and $V_{||}$ denotes that only the dimensions parallel to **w** suffer this compressional deformation.[a]

*a. On the independent status of* $\underline{\underline{w}}d^1$.

By virtue of the way that it is derived, $\underline{\underline{w}}d^1$ must be regarded as being both a *fundamental* and a totally *independent* component of the $\in_m$*-field-particle coupling physics* that is covered by **w-**gauge theory. This fundamental nature of $\underline{\underline{w}}d^1$ is qualitatively explained in Section IX where $\underline{\underline{w}}d^1$ is shown to be mathematically analogous to the '*solid-mass-energy*'–'*fluid-mass-energy*' coupling physics which initially prohibited *faster than sound* air travel in the mid 20th century—until it was theoretically understood that the linear physics of *soundspeed* and the nonlinear physics of *shockwaves* are inextricably tied to a *fundamental intensive property* of the medium, called *compressibility.*

2. $\underline{\underline{w}}d^2$: The dependent **w-**reduction of $\delta\mathcal{E}_m$ oscillation frequencies.

It is first shown that the GRV of electromagnetic radiation in $\grave{\underline{\Sigma}}$ causes the frequency ($\grave{\underline{f}}$) of an *optical-oscillator* ($\underline{S}$) to be reduced by an orientation dependent factor ranging from $f^0/\underline{\underline{\ell}}$ to $f^0/\underline{\underline{\ell}}^2$ as the orientation of $\underline{S}$ (relative to the $\grave{\underline{\mathbf{u}}}$ field in $\grave{\underline{\Sigma}}$) changes from perpendicular to parallel. And it then shown that the application of $\underline{\underline{w}}d^1$ removes this orientation sensitivity, completely—causing $\underline{S}$ to have a frequency, $\grave{\underline{f}} = f^0/\underline{\underline{\ell}}$, which is independent of its orientation relative to $\grave{\underline{\mathbf{u}}}$.

It is then asserted that the frequency of every $\delta\mathcal{E}_m$ oscillator must be reduced by the same $1/\underline{\underline{\ell}}$ factor in order to preserve the Null-$\underline{\mathbf{p}}_m$ Axiom. The $\underline{\underline{w}}d^2$ component of **w-**gauge theory is then defined by the requirement that $\underline{\underline{w}}d^2 \equiv f^0/f = \underline{\underline{\ell}} = \ell$ applies to any $\delta\mathcal{E}_m$ oscillator. This **w**−deformation is also defined by the equivalent requirement that $\underline{\underline{w}}d^2 = T/T^0 = \ell$ where T is time period defined by any $\delta\mathcal{E}_m$ process.

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[a] Note that the $\underline{\underline{w}}d^i$'s are generally derived within $\grave{\underline{\Sigma}}$ and then transformed into $\Sigma$ parameters via the Galilean relations of (2.4).

*a. On the dependent nature of* $\underline{\underline{w}}d^2$ .

From the way in which it is derived, it will be clear that $\underline{\underline{w}}d^2$ must be regarded as being a totally *dependent* component of the $\in_m$-field-particle coupling physics covered by **w-**gauge theory. Hence, different components of the $\underline{\underline{w}}d^i$ sub-group; $\underline{\underline{w}}d^{1,3-5}$, can then be invoked, as logic dictates, to explain the $\underline{\underline{w}}d^2$ deformations associated with different $\delta\mathcal{E}_m$ processes involving different $\delta\mathcal{E}_m$ particle interaction laws. Two other examples of the dependent nature of $\underline{\underline{w}}d^2$ are given via two independent ways that the mass deformation, $\underline{\underline{w}}d^4$, is derived in Sections III.J,K.

3. $\underline{\underline{w}}d^3$: The asimultaneity indigenous to a `$\underline{\mathbf{p}}_m$ field.

By examining the influence in $\underline{\Sigma}$ of `$\underline{\mathbf{u}}$ on the measurement of a $\delta\mathcal{E}_m$ particle's GRV via the *time of flight* (TOF) method, we deduce that such measurements would allow the Null-$\underline{\mathbf{p}}_m$ Axiom to be violated unless the TOF includes, from the perspective of `$\underline{\Sigma}$; not just the usual GRS-defined transport time, d`$\underline{x}^0$ = d`$\underline{\mathbf{r}}$/`$\underline{\beta}$, but also an ***asimultaneity** time*, $\delta$`$\underline{x}^0$ = `$\underline{\Gamma}^2$(`$\underline{\mathbf{u}}$•d`$\underline{\mathbf{r}}$) ≡ $\underline{\underline{w}}d^3$, which is indigenous to the `$\underline{\mathbf{p}}_m$ field and which gets physically transferred to, or mapped to, a pair of TOF clocks by means of, what appears in $\underline{\Sigma}$ to be, a *geodynamically symmetric synchronization* (GSS) of the clocks.

We then show that the measure of {(d`$\underline{x}^0$ + $\delta$`$\underline{x}^0$) ≡ D(`$\underline{x}^0$)} via the slow running TOF clocks in $\underline{\Sigma}$—as {d$\underline{x}^0$ = D(`$\underline{x}^0$)/`$\underline{\Gamma}$}—represents a generalization of $\underline{\underline{w}}d^2$ (from two events at the same point, to two events at different points) which both constitutes and explains *the temporal component of a general Lorentz transformation*. We thus show that taken together, $\underline{\underline{w}}d^{1-3}$ can be truthfully said to cause the d$\underline{x}^i$ of $\underline{\Sigma}$ to be related to the d$x^i$ of $\Sigma$ by a general Lorentz transformation. And we then use this result to develop transformations for the $\gamma$'s and $\beta$'s between $\Sigma$, $\underline{\Sigma}$, and `$\underline{\Sigma}$, which are then gainfully employed to derive $\underline{\underline{w}}d^{4-5}$.

4. $\underline{\underline{w}}d^4$: The propagational increase of $p^\pi$ mass.

Given $\underline{\underline{w}}d^{1\text{-}3}$ and the Null-$\underline{\mathbf{p}}_m$ Axiom we derive the need for a propagational increase in the mass of a $p^\pi$ particle in two ways: A rather trivial idealistic way—employing a freely spinning disc. And a more sophisticated practical way—employing a hypothetical experiment designed to generate zero-momentum pairs of $p^\pi$ particles. Both ways yield $\underline{\underline{w}}d^4$ in the form $\underline{\underline{w}}d^4$ = m/m$^0$ = $\ell$. And in support of III.D.2.a, it is noted that $\underline{\underline{w}}d^4$ and the conservation law of angular momentum are entirely sufficient to explain the $\underline{\underline{w}}d^2$ deformation of the time period defined by one rotation of a freely spinning disc.

5. $\underline{\underline{w}}d^5$: The **w-**variation of $p^\pi$ energy.

Given $\underline{\underline{w}}d^4$, we then derive $\underline{\underline{w}}d^5$ = E/E$^0$ = mc$^2$/m$^0$c$^2$ = $\underline{\underline{w}}d^4$ = $\underline{\underline{\ell}}$ = $\ell$—and thus E = mc$^2$—directly from the Newtonian law, dE = **F**•d**r**, for the change in $p^\pi$ energy (E), where **F** = d**p**/dt and **p** = mc$\beta$ = m$^0$c$\gamma\beta$**.**

### 6. Summary of $\underline{\underline{w}}d^{i}$'s

In the ways just outlined we will then obtain four **w**−deformations of **w**−gauge theory:

$$\underline{\underline{w}}d^1 = \underline{\underline{w}}d^2 = \underline{\underline{w}}d^4 = \underline{\underline{w}}d^5$$

$$V^0/V_{||} = f^0/f = mc^2/m^0c^2 = E/E^0 = \ell \equiv 1/[1 - \mathbf{w}^2]^{1/2}, \tag{3.54}$$

arbitrarily referenced to Σ, plus the fundamental *asimultaneity* component

$$\underline{\underline{w}}d^3 \equiv \delta\grave{}\underline{x}^0 \equiv \grave{}\underline{\Gamma}^2(\grave{}\underline{\mathbf{u}}\bullet d\grave{}\underline{\mathbf{r}}) \text{ light-seconds}, \tag{3.32d}$$

with $\underline{\underline{w}}d^{1-3}$ causing the $d\underline{x}^i$ of $\underline{\Sigma}$ to be related to the $dx^i$ of Σ by a general Lorentz transformation.

From Eqs. (3.54) it follows that $E = mc^2$ is a fundamental characteristic of the $\delta\mathcal{E}_m$ particle coupling to the ${}^0\in_m$-continuum which leaves the $\grave{}\underline{\mathbf{p}}_m(\grave{}\underline{x}^i)$ gauge-field in $\grave{}\underline{\Sigma}$ locally unobservable in $\underline{\Sigma}$, and thus, the $\boldsymbol{\epsilon}_m$ environment in $\underline{\Sigma}$ *empirically isotropic.* Hence, by expanding the laws governing $\delta\mathcal{E}_m$ interactions to include the GRV formula and $\underline{\underline{w}}d^{1-5}$, it is easy to understand how they are consistent with Maxwell's gravity theorem, and thus, how every $\underline{\Sigma}$ can be empirically equivalent to Σ.

It will thus become clear that **w-**gauge theory covers the relativity theory that Einstein derived—nearly 100 years earlier—from his exceedingly advanced *symmetry and simultaneity rooted arguments.* The correctness of which promoted revolutionary advances in theoretical physics—leading, eventually, to the conceptually sophisticated *gauge theory* which allowed the fundamental *symmetry and simultaneity* issues addressed by Einstein (and held to be consistent with ${}^0\in_m = 0$) to be covered by a *fundamental gauge-field theory* consistent with Maxwell's gravity theorem (and hence with ${}^0\in_m >> 0$) and thus the very kind of classical field theory and cosmology that Einstein had hoped to uncover with his personal *Maxwellian Program* (see Section IV).

### E. The **w**−*deformation* of $\delta\mathcal{E}_m$ 3-volume ≡ $\underline{\underline{w}}d^1$.

#### 1. The solitonal electromagnetic fields in Σ and $\grave{}\underline{\Sigma}$ sourced by a charge Q at $\underline{O}$ of $\underline{\Sigma}$.

##### *a. On the newly recognized fundamentality of* $\varphi_e$ *and* **A***, and *completeness of G3.*

By the middle of the 20th century, it had become apparent that Maxwell's theory of electrodynamics was not altered in any fundamental way by relativity theory. Consequently it became clear that the descriptions of classical radiation theory—most of which were derived before 1905—possessed a much more general validity than could have been originally supposed. Revealing that:

> "*All the detailed calculations of fields from moving charges made on the assumption that there is one frame in which the wave equation is correct are equivalent to those resulting from a covariant formulation of electrodynamics if we thus re-interpret the velocity.*" [13,a]

And by 1964 it had also become apparent that the pair of intrinsically unobservable electromagnetic fields, $\varphi_e$ and $\mathbf{A}^* = c\mathbf{A} = \beta\varphi_e$, associated with the existence and the propagation velocity of electric charge, are more fundamental than the measurable **E** and **B** fields representing various time and 3-space

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[a] Via w-gauge theory it will be seen that the same thing is true of *all* the (1 + 3) laws of fields and particles: They are all valid with respect to Σ, if we simply reinterpret a particle's velocity as its generalized rectilinear velocity $\beta = (\mathbf{u} + \mathbf{w})$, and include in the those laws the empirically confirmable gauge-deformations of **w** being here derived.

derivatives of $\varphi_e$ and $\mathbf{A}^*$.[14] The fields $\varphi_e$ and $\mathbf{A}^*$ are conveniently packaged here in (1 + 3) notation as $\varphi_e^i(x^i) = (\varphi_e, \mathbf{A}^*) = \varphi_e(x^i)(1, \underline{\underline{\beta}}) \equiv \varphi_e\underline{\underline{\beta}}^i$.[a]

*b. The wave equation governing the 3-dimensional distributions of* $\varphi_e$ *and* $\mathbf{A}^*$ *in* ${}^0\mathcal{E}_m$.

We begin the derivations of the $\underline{\underline{w}}d^i$ by deriving, from the perspectives of both $\Sigma$ and $`\underline{\Sigma}$, the 3–dimensional distributions of the fundamental soliton-like $\psi^i$ fields that envelop a small volume of electric charge $Q = \int\rho dV$ at the origin $\underline{O}$ of $\underline{\Sigma}$. The $\psi^i$ fields are then sourced by the presence of Q and by the $\underline{\underline{\mathbf{w}}} = \underline{\underline{\beta}}$ of Q in accordance with the following inhomogeneous wave equation of G3:

$$\Box\psi^i \equiv [\partial^2/\partial x^2 + \partial^2/\partial y^2 + \partial^2/\partial z^2 - \partial^2/(\partial x^0)^2]\psi^i = -\rho\underline{\underline{\beta}}^i(x, y, z)/\epsilon_0, \tag{3.2}$$

where $\underline{\underline{\beta}}^i = (1, \underline{\underline{\beta}})$, and the $x^\alpha$ have been expanded into the set of Cartesian coordinates x, y, and z, for descriptive convenience.

*c. Simplified formulas which exploit the constancy of* $\underline{\underline{\beta}}^i$.

In view of the constancy of $\underline{\underline{\beta}}^i$, $\Box\psi^i = \Box(\varphi_e\underline{\underline{\beta}}^i) = \underline{\underline{\beta}}^i\,\Box\varphi_e$. Hence Eq. (3.2) is completely equivalent to the simple scalar wave equation

$$\Box\varphi_e = -\rho(x, y, z)/\epsilon_0 \tag{3.3}$$

for $\varphi_e$ only. The solution of which, $\varphi_e(x^i)$, then yields the solution $\mathbf{A}^*(x^i) = \varphi_e(x^i)\beta(x^i)$ as well.

*d. The inherent axial symmetry of* $\psi^i$ *and* $`\underline{\psi}^i$.

With no loss of generality; $\underline{\underline{\beta}}$ is defined to be along the co-aligned positive z-axes of $\Sigma$, $\underline{\Sigma}$ and $`\underline{\Sigma}$. And since $\underline{\underline{\beta}} = \underline{\underline{\beta}}\mathbf{e}_z$ is constant with time, we can predict that $\varphi_e$, when explicitly referenced to $`\underline{\Sigma}$, as $`\underline{\varphi}_e$, will be a temporally-constant, axially-symmetric, soliton-like structure that envelops and co–propagates with Q. It follows that any $\partial\varphi_e/\partial x^0$ relative to a fixed point in $\Sigma$ must be due entirely the constant $\underline{\underline{\mathbf{w}}}$ of Q and/or of $`\underline{\varphi}_e$. The formula describing this $\mathbf{w}$–induced time rate of change is derived as follows:

*e. The explicit time-rate-of-change of* $\varphi_e(x^i)$ *due entirely to* $\underline{\underline{\mathbf{w}}}(Q)$.

Equations (3.2) subsume the continuity equation $\nabla\bullet\mathbf{A}^* + \partial\varphi_e/\partial x^0 = 0$. And $\mathbf{A}^* = \underline{\underline{\beta}}\varphi_e$. Therefore, by inserting the latter equality into the former equality, and using $\underline{\underline{\beta}}(\nabla\bullet\mathbf{A}^*) = (\underline{\underline{\beta}}\bullet\nabla)\mathbf{A}^*$, we obtain

$$(\underline{\underline{\beta}}\bullet\nabla + \partial/\partial x^0)\varphi_e = 0, \tag{3.4}$$

as a special form of the continuity equation consistent with a constant $\underline{\underline{\beta}}$ and $\mathbf{A}^* = \underline{\underline{\beta}}\varphi_e$. The explicit time rate of change of $\varphi_e$ referenced to $\Sigma$ is then given by

$$\partial\varphi_e/\partial x^0 = -(\underline{\underline{\beta}}\bullet\nabla)\varphi_e(x^\alpha) = -\underline{\underline{\beta}}\partial\varphi_e/\partial z. \tag{3.5}$$

*f. The resulting wave equation for* $\varphi_e$ in $\Sigma$.

Using Eq. (3.5) to eliminate $\partial\varphi_e/\partial x^0$ in Eq. (3.3) then yields

$$[\partial^2/\partial x^2 + \partial^2/\partial y^2 + (1 - \underline{\underline{\beta}}^2)\partial^2/\partial z^2]\varphi_e = -\rho(x, y, z)/\epsilon_0, \tag{3.6}$$

referenced to $\Sigma$.

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[a] A (1 + 3) notation is used throughout (instead of the traditional (3 + 1) notation) because it facilitates a smoother transition to the (1 + *p*)-dimensional notation appropriate for hyper-energy theory.

*g. The complementary $\varphi_e(x^i)$ and $\grave{\underline{\varphi}}_e(\grave{\underline{x}}^i)$ solutions of (3.6a).*

The solution of (3.6a) will yield the scalar electromagnetic potential $\varphi_e(x^0, \mathbf{r})$ that *arrives* at a given fixed point **(r)** in $\Sigma$ at the instant $(x^0)$ that the charge Q, propagating with velocity $\underline{\underline{\beta}}$, *arrives* at the point $\underline{\underline{\mathbf{r}}} = \underline{\underline{\beta}}x^0$ in $\Sigma$. These two events in $\Sigma$ will therefore be simultaneous in $\Sigma$ with a spatial separation, $\grave{\underline{\mathbf{r}}} = \mathbf{r} - \underline{\underline{\mathbf{r}}}$, that changes with time in accordance with the formula $\grave{\underline{\mathbf{r}}} = \mathbf{r} - \underline{\underline{\beta}}x^0$.

With respect to $\Sigma$ then, the solution of (3.6a) will have a non steady, time-varying, axially symmetric form, $\varphi_e = \varphi_e(Q, r, \theta, x^0, \underline{\underline{\beta}})$ that gives rise to a similarly structured and varying vector potential, $\mathbf{A}^* = c\mathbf{A} = \underline{\underline{\beta}}\varphi_e$, and, their corresponding effects; as observable $\mathbf{E}(x^0)$ and $\mathbf{B}(x^0)$ fields at **r**.

But the solution of (3.6a) can also be conveniently expressed with respect to the abstractly co–moving and co-aligned reference system $\grave{\underline{\Sigma}}$ to provide a steady, non time-varying, axially symmetric description of the potential; as $\grave{\underline{\varphi}}_e = \grave{\underline{\varphi}}_e(Q, \grave{\underline{\mathbf{r}}} = \grave{\underline{\mathbf{n}}}\grave{\underline{r}}, \grave{\underline{\theta}}, \grave{\underline{\mathbf{u}}})$. Thus, $\grave{\underline{\varphi}}_e$ can be regarded as the solution to the equivalent wave equation

$$[\, \partial^2/\partial\grave{\underline{x}}^2 + \partial^2/\partial\grave{\underline{y}}^2 + (1 - \grave{\underline{\mathbf{u}}}^2)\partial^2/\partial\grave{\underline{z}}^2 \,]\grave{\underline{\varphi}}_e = -\grave{\underline{\rho}}(\grave{\underline{x}}, \grave{\underline{y}}, \grave{\underline{z}})/\epsilon_0, \tag{3.7}$$

referenced to $\grave{\underline{\Sigma}}$. Thereby providing an intuitive *in situ* view of how a local $\grave{\mathbf{p}}_m$–field couples to and alters the $\delta\mathcal{E}_m$ field of electric charge. A view which can be easily transformed into $\varphi_e = \varphi_e(Q, r, \theta, x^0, \underline{\underline{\beta}})$ by simply setting $\grave{\underline{r}} = |\,\mathbf{r} - \underline{\underline{\beta}}x^0|$ and $\sin\grave{\underline{\theta}} = (r\sin\theta)/\grave{\underline{r}}$.

Because the $\grave{\underline{\Sigma}}$ view of $Q\text{-}{}^0\in_m$ coupling happens to be entirely consistent with the restricted (constant $\underline{\underline{\beta}}$) form of the well known Liénard-Wiechert potentials, we will describe the solution of (3.7) first. With due allowance for obvious differences in notation, it has been argued[15] that the solution of (3.7) is the restricted (constant $\grave{\underline{\mathbf{u}}}$) Liénard-Wiechert potential

$$\grave{\underline{\varphi}}_e = \frac{1}{4\pi\in_0}\int\frac{\grave{\underline{\rho}}(\grave{\underline{\mathring{x}}}^\alpha)d\grave{\underline{\mathring{V}}}}{\grave{\underline{\lambda}}} \cong \frac{Q}{4\pi\in_0\grave{\underline{S}}}, \tag{3.8}$$

where

$$\grave{\underline{\lambda}} = \{\, [(\grave{\underline{x}} - \grave{\underline{\mathring{x}}})^2 + (\grave{\underline{y}} - \grave{\underline{\mathring{y}}})^2](1 - \grave{\underline{\mathbf{u}}}^2) + (\grave{\underline{z}} - \grave{\underline{\mathring{z}}})^2 \}^{1/2}, \tag{3.9a}$$

and

$$\grave{\underline{S}} = \{\, [\grave{\underline{x}}^2 + \grave{\underline{y}}^2](1 - \grave{\underline{\mathbf{u}}}^2) + \grave{\underline{z}}^2 \}^{1/2} \text{ for a point charge approximation,} \tag{3.9b}$$

$$= \grave{\underline{r}}\,[1 - (\grave{\underline{\mathbf{n}}} \times \grave{\underline{\mathbf{u}}})^2]^{1/2}, \tag{3.9c}$$

$$= \grave{\underline{r}}\,[1 - \grave{\underline{\mathbf{u}}}^2 \sin^2\grave{\underline{\theta}}]^{1/2}. \tag{3.9d}$$

Equations (3.7) clearly display the *axial symmetry* and the *rectilinear asymmetry* of $\grave{\underline{\varphi}}_e$ and of $\grave{\underline{\mathbf{A}}}^* = -\grave{\underline{\mathbf{u}}}\grave{\underline{\varphi}}_e = \mathbf{A}^*$ in relation to the relative $\mathbf{p}_m$–field, $\grave{\underline{\mathbf{u}}}$. And because of the constancy of $\underline{\underline{\beta}}$, this $\grave{\underline{\Sigma}}$ perception of the solitonal structure of $\grave{\underline{\varphi}}_e$ is wholly un-encumbered by the relations; which otherwise need to be taken into account, between the *present* and *retarded* position and GRV of Q. And transforming to the perspective of $\Sigma$ in the manner described above gives

$$\varphi_e(Q, r, \theta, x^0, \underline{\underline{\beta}}) = \frac{Q}{4\pi\in_0[r^2(1 - \underline{\underline{\beta}}^2\sin^2\theta) + (\underline{\underline{\beta}}x^0)^2 - 2\mathbf{r}\bullet\underline{\underline{\beta}}x^0]^{1/2}}. \tag{3.10}$$

The solitonal electric and magnetic fields of $\varphi_e$ and $\grave{\underline{\varphi}}_e$ are described next.

*h. The solitonal electric and magnetic fields of* $\varphi_e$ *and* $`\underline{\varphi}_e$.

For the point charge approximation, and with due allowance for obvious differences in notation, the solitonal electric and magnetic fields of $\varphi_e$ are given by formulas

$$\mathbf{E}(Q, \mathbf{r}, \theta, x^0, \underline{\underline{\beta}}) = -\nabla(\varphi_e) - \partial\mathbf{A}^*/\partial x^0 = -(\nabla - \underline{\underline{\beta}}\bullet\nabla]\varphi_e, \tag{3.11a}$$

$$= \frac{Q(1-\underline{\underline{\beta}}^2)}{4\pi\epsilon_0}\frac{(\mathbf{r}-\underline{\underline{\beta}}x^0)}{[r^2(1-\underline{\underline{\beta}}^2\sin^2\theta) + (\underline{\underline{\beta}}x^0)^2 - 2\mathbf{r}\bullet\underline{\underline{\beta}}x^0]^{3/2}}, \tag{3.11b}$$

$$\mathbf{B}c(Q, \mathbf{r}, \theta, x^0, \underline{\underline{\beta}}) = \nabla\times\mathbf{A}^* = -\nabla\times(\varphi_e\mathbf{u}) = Q\,\mathbf{u}\times\nabla(\varphi_e), \tag{3.11c}$$

$$= \frac{Q(1-\underline{\underline{\beta}}^2)}{4\pi\epsilon_0}\frac{\underline{\underline{\beta}}\times\mathbf{r}}{[r^2(1-\underline{\underline{\beta}}^2\sin^2\theta) + (\underline{\underline{\beta}}x^0)^2 - 2\mathbf{r}\bullet\underline{\underline{\beta}}x^0]^{3/2}} = \underline{\underline{\beta}}\times\mathbf{E}, \tag{3.11d}$$

while the solitonal electric and magnetic fields of $`\underline{\varphi}_e$ are given by better known formulas [16,17,18]

$$`\underline{\mathbf{E}}(Q, `\underline{\mathbf{r}}, `\underline{\theta}, `\underline{\mathbf{u}}) = -`\underline{\nabla}(`\underline{\varphi}_e) - \partial`\underline{\mathbf{A}}^*/\partial`\underline{x}^0 = -(`\underline{\nabla} + `\underline{\mathbf{u}}\bullet`\underline{\nabla}]`\underline{\varphi}_e, \tag{3.12a}$$

$$= \frac{Q(1-`u^2)}{4\pi\epsilon_0}\frac{`\underline{\mathbf{r}}}{`\underline{r}^3\,[1-`\underline{\mathbf{u}}^2\sin^2`\underline{\theta}]^{3/2}}, \tag{3.12b}$$

$$`\underline{\mathbf{B}}c(Q, `\underline{\mathbf{r}}, `\underline{\theta}, `\underline{\mathbf{u}}) = `\underline{\nabla}\times`\underline{\mathbf{A}}^* = -`\underline{\nabla}\times(`\underline{\varphi}_e`\underline{\mathbf{u}}) = Q\,`\underline{\mathbf{u}}\times`\underline{\nabla}(`\underline{\varphi}_e), \tag{3.12c}$$

$$= \frac{Q(1-`\underline{\mathbf{u}}^2)}{4\pi\epsilon_0}\frac{`\underline{\mathbf{r}}\times`\underline{\mathbf{u}}}{`\underline{r}^3\,[1-`\underline{\mathbf{u}}^2\sin^2`\underline{\theta}]^{3/2}} = `\underline{\mathbf{E}}\times`\underline{\mathbf{u}}. \tag{3.12d}$$

Because these effects of $Q\text{–}{}^0\epsilon_m$ coupling have long been empirically confirmed by the successful application of the theory of electromagnetic radiation, there can be no question as to the reality of these solitonal electromagnetic fields, and thus, no question regarding the revelation that the solitonal $`\underline{\mathbf{E}}$, and $`\underline{\mathbf{B}}$ fields in $`\underline{\Sigma}$ may be regarded as being induced and sustained by the interaction of the steady $`\mathbf{p}_m$–field $`\underline{\mathbf{u}}$ with the steady field of Q.

But these solitonal fields contain first and second order effects of $`\underline{\mathbf{u}}$ which could, in principle, allow $`\underline{\mathbf{u}}$ to be measured in $\underline{\Sigma}$, and thus, the Null-$\underline{\mathbf{p}}_m$ Axiom to be violated via an electro-mechanical $`\underline{\mathbf{u}}$-sensor of the Trouton-Noble type. Hence, there is no question about the necessity for a **w**–deformation that will cause the *true* solitonal electromagnetic fields of Q—in both Σ and $`\underline{\Sigma}$—to be consistent with an *empirical* Coulomb potential and electric field of Q in $\underline{\Sigma}$ satisfying $\underline{\psi}^i = (\underline{\varphi}_e, \mathbf{0})$, where $\underline{\varphi}_e = Q/(4\pi\epsilon_0\underline{\mathbf{r}})$.

It follows that in order to preserve Maxwell's gravity theorem and the reality of the solitonal $`\underline{\psi}^i$, $`\underline{\mathbf{E}}$, and $`\underline{\mathbf{B}}$ fields which envelop Q as co-propagating disturbances in the ${}^0\epsilon_m$–continuum, we need only derive a **w-**gauge deformation that would cause $\underline{\varphi}_e = Q/(4\pi\epsilon_0\underline{r})$ to be empirically con-sistent with $`\underline{\varphi}_e = Q/(4\pi\epsilon_0`\underline{s})$, so that for this experimental situation, in particular, $`\underline{\mathbf{u}} \neq \mathbf{0}$ will be consistent with $\underline{\mathbf{u}} = \mathbf{0}$. The needed **w**–deformation, labeled as $\underline{\underline{w}}d^1$, is derived as follows.

2. The structure of $\underline{\underline{w}}d^1$.

With due allowances for mere differences in notation, it has been pointed out[19] that the simple change of variables:

$$\underline{x} = \grave{}\underline{x}, \tag{3.13a}$$
$$\underline{y} = \grave{}\underline{y}, \tag{3.13b}$$
$$\underline{z} = \grave{}\underline{\Gamma}\,\grave{}\underline{z} = \grave{}\underline{\Gamma}\,(z - \underline{\underline{\beta}}x^0), \tag{3.13c}$$

serves to transform (3.7) into a simple electrostatic Poison equation,

$$( \partial^2/\partial\underline{x}^2 + \partial^2/\partial\underline{y}^2 + \partial^2/\partial\underline{z}^2 )\ \underline{\varphi}_e = -\grave{}\underline{\Gamma}\ \underline{\rho}(\underline{x}, \underline{y}, \underline{z}/\grave{}\underline{\Gamma})/\epsilon_0, \tag{3.14}$$

of which the solution is the ordinary Coulomb potential

$$\underline{\varphi}_e(\underline{\mathbf{r}}) = \frac{1}{4\pi\epsilon_0}\int\frac{\underline{\rho}(\mathring{\underline{x}}/\grave{}\underline{\Gamma}, \mathring{\underline{y}}, \mathring{\underline{z}})d\mathring{\underline{V}}(\mathring{\underline{x}}/\grave{}\underline{\Gamma}, \mathring{\underline{y}}, \mathring{\underline{z}})}{|\underline{\mathbf{r}} - \mathring{\underline{\mathbf{r}}}|}, \tag{3.15a}$$

where

$$|\underline{\mathbf{r}} - \mathring{\underline{\mathbf{r}}}| = [\, (\underline{x} - \mathring{\underline{x}})^2 + (\underline{y} - \mathring{\underline{y}})^2 + (\underline{z} - \mathring{\underline{z}})^2\,]^{1/2}, \tag{3.15b}$$

which reduces to

$$\underline{\varphi}_e(\underline{\mathbf{r}}) = \frac{Q}{4\pi\epsilon_0\,\underline{r}} \tag{3.16}$$

for a point-charge approximation, where $\underline{r} = [\underline{x}^2 + \underline{y}^2 + \underline{z}^2\,]^{1/2}$.

From this we conclude that, in order for the Null-$\underline{\mathbf{p}}_m$ Axiom to be preserved despite the rectilinear asymmetry of the solitonal structure of the electromagnetic field that envelops a uniformly propagating point electric charge, the 3-volume (V) of any $\delta\mathcal{E}_m$ structure must suffer a compression solely along the direction of its propagation by the factor $\grave{}\underline{V}_{\|}/V^0 = 1/\grave{}\underline{\Gamma} \equiv 1/\underline{\underline{w}}d^1$, thereby causing the 3-dimensional distance coordinates of $\underline{\Sigma}$ and $\grave{}\underline{\Sigma}$ to be related by Eqs. (3.13), and the lengths of the corresponding radius vectors in $\underline{\Sigma}$ and $\grave{}\underline{\Sigma}$ (and their changes) to be related by the formulas

$$\underline{r} = \grave{}\underline{\Gamma}[\grave{}\underline{\mathbf{r}}^2 - (\grave{}\underline{\mathbf{r}}\times\grave{}\underline{\mathbf{u}})^2]^{1/2} \equiv \grave{}\underline{\Gamma}\,\grave{}\mathfrak{S}. \tag{3.17a}$$

$$d\underline{\mathbf{r}} = d\underline{\mathbf{r}}_{\|} + d\underline{\mathbf{r}}_{\perp} = (\grave{}\underline{\Gamma})d\grave{}\underline{\mathbf{r}}_{\|} + d\grave{}\underline{\mathbf{r}}_{\perp} \equiv (\grave{}\underline{\Gamma})d\grave{}\boldsymbol{\mathfrak{S}}. \tag{3.17b}$$

$$d\underline{r} = \grave{}\underline{\Gamma}[1 - (\grave{}\underline{\mathbf{n}}\times\grave{}\underline{\mathbf{u}})^2]^{1/2}\,d\grave{}\underline{r} \equiv \grave{}\underline{\Gamma}d\grave{}\mathfrak{S}, \tag{3.17c}$$

$$= [\,(\grave{}\underline{\Gamma}d\grave{}\underline{\mathbf{r}}_{\|})^2 + (d\grave{}\underline{\mathbf{r}}_{\perp})^2\,]^{1/2}. \tag{3.17d}$$

Using the Galilean coordinate relation, $d\grave{}\underline{\mathbf{r}} = (d\mathbf{r} - d\underline{\mathbf{r}})$, in (3.17d) gives

$$d\underline{r} = \{[\underline{\gamma}(d\mathbf{r}_{\|} - d\underline{\mathbf{r}})]^2 + (d\mathbf{r}_{\perp})^2\,\}^{1/2}, \tag{3.17e}$$

and hence,

$$d\underline{\mathbf{r}} = d\underline{\mathbf{r}}_{\|} + d\underline{\mathbf{r}}_{\perp} = \underline{\gamma}(d\mathbf{r}_{\|} - d\underline{\mathbf{r}}) + d\mathbf{r}_{\perp}. \tag{3.17f}$$

Thereby showing that $\underline{\underline{w}}d^1$ can be formally regarded as describing the spatial component of a general Lorentz transformation.

The $\underline{\underline{w}}d^1$ deformation of **w-**gauge theory, arbitrarily referenced to Σ for an arbitrary $\delta\mathcal{E}_m$ structure, is then conveniently defined as

$$\underline{\underline{w}}d^1 = V^0/V_{\|} = \underline{\underline{\ell}} = \ell. \tag{3.18}$$

*a. The implied structure of $\underline{\underline{w}}d^5$.*

Using (3.17a) to eliminate $\underline{r}$ in (3.16), and comparing the result to Eqs. (3.8) reveals that

$$`\underline{\varphi}_e = `\underline{\Gamma}\, \underline{\varphi}_e, \tag{3.19a}$$

and, equivalently, that

$$\varphi_e = \underline{\underline{\gamma}}\, \underline{\varphi}_e, \tag{3.19b}$$

$$= \underline{\underline{\ell}}\, \underline{\varphi}_e, \text{ most generally.} \tag{3.19c}$$

Since $\varphi_e$ moderates the energy of interacting charges, this result suggests that the interaction energy of charged particles increases nonlinearly with propagation velocity. But this could lead to a violation of the Null-$\underline{\mathbf{p}}_m$ Axiom if it did not apply to every conceivable type of particle-interaction energy. Hence, what we have here is a hint of the fifth **w**–deformation of **w-**gauge theory to be derived in the more general form $\underline{\underline{w}}d^5 = E/E^0 = \underline{\underline{\ell}}$.

*b. The independent nature of $\underline{\underline{w}}d^1$.*

The way in which $\underline{\underline{w}}d^1$ was derived makes it clear that $\underline{\underline{w}}d^1$ is an isolated effect of the 100% coupling of ${}^0\in_m$ and its $\delta\mathcal{E}_m$ structures, completely independent of the three other **w**–deformations.

F. The general asymmetry of particle transport in $`\underline{\Sigma}(`\underline{x}^i)$.

1. The generalized rectilinear speed (GRS) of an arbitrary $\delta\mathcal{E}_m$ particle in $`\Sigma(`x^i)$.

Squaring $`\underline{\mathbf{w}} = (`\underline{\beta} - `\underline{\mathbf{u}})$, solving for $`\underline{\beta} = `\underline{\mathbf{n}}\bullet`\underline{\beta}$, and using the fact that $`\underline{\mathbf{w}} = \mathbf{w} = \beta$ (derived in III.A.1.a), yields the following three equivalent expressions;

$$`\underline{\beta} = [\, `\underline{\mathbf{w}}^2 - (`\underline{\mathbf{n}}\times`\underline{\mathbf{u}})^2\,]^{½} + `\underline{\mathbf{n}}\bullet`\underline{\mathbf{u}}, \tag{3.20a}$$

$$= [\ \beta^2 - (`\underline{\mathbf{n}}\times\underline{\underline{\beta}})^2\ ]^{½} - `\underline{\mathbf{n}}\bullet\underline{\underline{\beta}}, \tag{3.20b}$$

$$= \quad `\underline{\mathbf{n}}\bullet\beta - `\underline{\mathbf{n}}\bullet\underline{\underline{\beta}}, \tag{3.20c}$$

for the *generalized coordinate speed* (GRS) of an arbitrary particle (p) with propagation velocity $`\underline{\mathbf{w}}$ in $`\underline{\Sigma}$, as a function of p's *transport direction* $`\underline{\mathbf{n}}$(p) and the $`\mathbf{p}_m$ field, $`\underline{\mathbf{u}} = `\underline{\mathbf{p}}_m/{}^0\in_m = -\,\underline{\underline{\mathbf{w}}} = -\underline{\underline{\beta}}$.

*a. Galilean invariant $\in_m$–particle coupling strengths.*

| | | | |
|---|---|---|---|
| With respect to: | $\Sigma$ | and | $`\underline{\Sigma}$: |
| The coupling strengths due to **w** and $\underline{\underline{\mathbf{w}}}$ are: | $\ell(\mathbf{w}^2) = \gamma(\beta^2)$ | $=$ | $`\underline{\ell}(`\underline{\mathbf{w}}^2)$, (3.21a) |
| | $\underline{\underline{\ell}}(\underline{\underline{\mathbf{w}}}^2) = \underline{\underline{\gamma}}(\underline{\underline{\beta}}^2)$ | $=$ | $`\underline{\Gamma}(`\underline{\mathbf{u}}^2)$. (3.21b) |

Hence, from Eq. (3.21a) it is seen that the coupling strength of a $p^\pi$ particle is invariant under a Galilean transformation of references systems. The fact that $`\underline{\Gamma}$ is conveniently employed for gauge-field derivations should not detract from the fact that the coupling strength of a $p^\pi$ particle is always expressed, most meaningfully and most generally, by $\ell(\mathbf{w}^2)$ where $\mathbf{w} = (\beta - \mathbf{u})$ has a triple degeneracy which, as is elaborated more fully in Section III.A, causes it to reduce to $\mathbf{w} = \beta$ in the *inertial* 3–space-energy continuum, to $\mathbf{w} = -\,\mathbf{u}$ at a fixed $x^\alpha$ of $\Sigma^d$ where matter-gravity is present, and to $\mathbf{0}$ in absolute ($\beta = \mathbf{u}$) gravitational free fall.

2. The GRS of electromagnetic radiation in $`\underline{\Sigma}(`\underline{x}^i)$.

The special case in which $`\underline{\mathbf{w}}(p)$ refers to the propagation of electromagnetic radiation or $p^0$ particles in $`\underline{\Sigma}(`\underline{x}^i)$ is then described by putting $`\underline{\mathbf{w}}^2 = (\mathbf{w}^0)^2 = 1$ in Eq. (3.20a), and by adding a zero superscript to the $`\underline{\beta}$ and $`\underline{\mathbf{n}}$ parameters, to give $`\underline{\beta}^0 = `\underline{\mathbf{u}} + `\underline{\mathbf{w}}^0$, and thus,

$$`\underline{\beta}^0 = `\underline{\beta}^0(`\underline{\mathbf{n}}^0,\ `\underline{\mathbf{u}}) = `\underline{\mathbf{n}}^0\bullet`\underline{\mathbf{u}} + [\,1 - (`\underline{\mathbf{n}}^0\times`\underline{\mathbf{u}})^2\,]^{1/2}. \tag{3.22a}$$

Solving for the particular values of $`\underline{\beta}^0$ when $`\underline{\beta}^0$ is parallel and perpendicular to $`\underline{\mathbf{u}}$ gives:

$$(`\underline{\beta}^0)_{\parallel} = 1 + `\underline{\mathbf{n}}^0\bullet`\underline{\mathbf{u}}, \tag{3.22b}$$

$$(`\underline{\beta}^0)_{\perp} = (1 - `\underline{\mathbf{u}}^2)^{1/2} \equiv 1/`\underline{\Gamma}, \tag{3.22c}$$

G. Derivation of $\underline{\underline{w}}d^2$ from the **w**-sensitivity of an optical oscillator's frequency.

Consider an optical oscillator, $\underline{S}$, in $\underline{\Sigma}$ whose optical length, frequency, and period referenced to $`\underline{\Sigma}$ is $`\underline{L} = `\underline{\mathbf{n}}\bullet`\underline{\mathbf{L}}$, $`\underline{f}$, and $`\underline{T} = 1/`\underline{f}$. The $`\underline{T}$ of $\underline{S}$ is fixed by the sum of the to–and–fro ($`\underline{\mathbf{n}}^0$ and $-`\underline{\mathbf{n}}^0$) transport times of electromagnetic radiation over $`\underline{L}$. Employing (3.22a), with $`\underline{\mathbf{n}}^0(p^0)$ restricted to $\pm\,`\underline{\mathbf{n}}$, yields

$$`\underline{T} = [`\underline{T}(`\underline{\mathbf{n}}, `\underline{\mathbf{u}}) + `\underline{T}(-`\underline{\mathbf{n}}, `\underline{\mathbf{u}})] = (2`\underline{L}/c)\,`\underline{\Gamma}^2\,[\,1 - (`\underline{\mathbf{n}}\times`\underline{\mathbf{u}})^2\,]^{1/2}, \tag{3.23}$$

where $`\underline{T}(`\underline{\mathbf{n}}, `\underline{\mathbf{u}}) = `\underline{L}/[c`\underline{\beta}^0(`\underline{\mathbf{n}},\ `\underline{\mathbf{u}})]$, and $`\underline{\beta}^0(`\underline{\mathbf{n}},\ `\underline{\mathbf{u}})$ is given by Eq. (3.22a).

This shows that, if $`\underline{L}$ was a constant unaffected by $`\underline{\mathbf{u}}$, the ${}^0\!\in_m$-$p^0$ coupling would cause the frequency of $\underline{S}$ to be reduced in an orientation ($`\underline{\mathbf{n}}$) sensitive manner—by a factor ranging from $1/`\underline{\Gamma}$ for $`\underline{\mathbf{L}}$ perpendicular to $`\underline{\mathbf{u}}$ (defining $`\underline{L}_{\perp}$), to $1/`\underline{\Gamma}^2$ for $`\underline{\mathbf{L}}$ parallel to $`\underline{\mathbf{u}}$ (defining $`\underline{L}_{\parallel}$)—which could, in principle, allow $`\underline{\mathbf{u}}$ to be measured and the Null-$\underline{\mathbf{p}}_m$ Axiom to be violated.

If one didn't know about $\underline{\underline{w}}d^1$, this would lead one to conclude that the ${}^0\!\in_m$-$\underline{S}$ coupling must gauge deform the physical geometry of $\underline{S}$ and $\underline{\Sigma}$ in a way that eliminates the orientation sensitivity of $`\underline{T}$ described by (3.23). Hence, we now prove that $\underline{\underline{w}}d^1$ fulfills this requirement of **w**-gauge theory.

From the scalar representation of $\underline{\underline{w}}d^1$:

$$\underline{r} = `\underline{\Gamma}[`\underline{\mathbf{r}}^2 - (`\underline{\mathbf{r}}\times`\underline{\mathbf{u}})^2]^{1/2} \equiv `\underline{\Gamma}`\mathfrak{s}, \tag{3.17a}$$

$$= `\underline{\Gamma}`\underline{r}\,[1 - (`\underline{\mathbf{n}}\times`\underline{\mathbf{u}})^2]^{1/2},$$

it follows that

$$`\underline{L} = \underline{L}/\{`\underline{\Gamma}\,[\,1 - (`\underline{\mathbf{n}}\times`\underline{\mathbf{u}})^2\,]^{1/2}\}, \tag{3.24a}$$

$$= L^0/\{`\underline{\Gamma}\,[\,1 - (`\underline{\mathbf{n}}\times`\underline{\mathbf{u}})^2\,]^{1/2}\}. \qquad 3.24b)$$

And substitution of (3.24b) in (3.23) then yields

$$`\underline{T} = `\underline{\Gamma}\,(2L^0/c) = `\underline{\Gamma}\,\underline{T} = `\underline{\Gamma}\,T^0, \tag{3.25}$$

or

$$`\underline{f} = 1/`\underline{T} = \underline{f}/`\underline{\Gamma} = f^0/`\underline{\Gamma}. \tag{3.26}$$

This shows that ${}^{0}\in_{m}$-$\underline{S}$ coupling, embracing the $\underline{\underline{w}}d^1$ component of **w-**gauge theory, causes the frequency of $\underline{S}$ to be reduced in a directionally invariant manner by the factor $1/\grave{}\underline{\Gamma}$; a fact which, in principle, could still be used to violate the Null-$\underline{\mathbf{p}}_m$ Axiom, and thus, the conservation law of rectilinear $\delta\mathcal{E}_m$-momentum, unless, the frequency and time measures defined by every type of $\delta\mathcal{E}_m$–oscillator suffer the same **w**–deformation as $\underline{S}$.

H. The definition of $\underline{\underline{w}}d^2$.

Generalizing the results described by Eqs. (3.25-6) to an arbitrary $\delta\mathcal{E}_m$ oscillator with proper extensive (integer cycle) parameters $f^0$ and $T^0$, we then define $\underline{\underline{w}}d^2$ with respect to $\Sigma$ as

$$\underline{\underline{w}}d^2 = T/T^0 = f^0/f = \underline{\underline{\ell}} = \ell. \tag{3.27}$$

1. The dependent nature of $\underline{\underline{w}}d^2$.

From the way in which $\underline{\underline{w}}d^2$ was derived, it is clear that this **w**–deformation is *a totally dependent consequence of ${}^{0}\in_{m}$–particle coupling*. It follows that the $\delta\mathcal{E}_m$ *time-dilation* associated with any given $\delta\mathcal{E}_m$ process can always be explained in terms of other—more fundamental—w–gauge deformations affecting the given $\delta\mathcal{E}_m$ process. In Section III.J.1, for instance, the necessity of $\underline{\underline{w}}d^2$ is derived simply from the need to conserve angular $\delta\mathcal{E}_m$ momentum in the presence of the more fundamental **w**–deformation of $\delta\mathcal{E}_m$ mass, $\underline{\underline{w}}d^4$, where $\underline{\underline{w}}d^4 = m/m_0 = \ell$.

*a. On the physical nature of time.*

Given $\underline{\underline{w}}d^2$, it follows that the fundamental nature and physics of time is to be found in the physics accounting for the existence of the ${}^{0}\in_{m}$-continuum, as the physical solution of Maxwell's 3–space-energy problem. And the next **w**–deformation will shed some light on this by allowing us to conclude that the $\mathbf{p}_m(x^i)$ field, in addition functioning as a 3–vector potential of matter gravity, is also—and perhaps more fundamentally—a broken symmetry of a $(1 + p)$-dimensional *time field*, $\mathbf{p}(x^a)$ which characterizes the compactified ${}^{0}\in_{m}$-continuum by the vanishing of its *3-space* components therein—something which is graphically illustrated in Figs. II–IV of Sections VII and VIII.

I. $\underline{\underline{w}}d^3$: The asimultaneity indigenous to a $\grave{}\underline{\mathbf{p}}_m$ field.

1. A geodynamically symmetric synchronization (GSS).

In $\Sigma$, a geometrodynamically symmetric synchronization (GSS) results when two synchronization signals—sent out simultaneously from the midpoint of two clocks at time $x^0$, with equal and opposite transport velocities, $\beta_s$—cause the times displayed by their respective target clocks to be reset (upon their respective impacts) to the same time—typically $[x^0 + dr/(2\beta_s)]$ light seconds, for a clock separation of dr.

A GSS (or its equivalent) fulfills the important practical need of replacing the generally unknown phase difference between two otherwise identical clocks with a phase difference that is known to be zero (or some other number which can be mathematically eliminated from the measured difference in time). And this must be equally true in the effectively isotropic environment of $\underline{\Sigma}$ as well.

However, on the basis of the $\underline{\underline{w}}d^{1-2}$ derivations and the perspective of $\grave{}\underline{\Sigma}$, it is reasonable to expect that a GSS in $\underline{\Sigma}$ will cause the clock times in $\underline{\Sigma}$ to actually differ by a specific, $\grave{}\underline{\mathbf{u}}$ and $d\grave{}\underline{\mathbf{r}}$ dependent amount, ($\delta\grave{}\underline{x}^0$, say) which can be said to quantify, from the perspective of $\grave{}\underline{\Sigma}$, a differential amount of *asimultaneity* between the two clock readings. But in accordance with the Null-$\underline{\mathbf{p}}_m$ Axiom, $\delta`\underline{x}^0$ must be rendered unobservable in $\underline{\Sigma}$ by the laws of $\delta\mathcal{E}_m$ particle physics.

The purpose of this section is then twofold: Formulate $\delta`\underline{x}^0$ mathematically, and then show how $\delta`\underline{x}^0$, as $\underline{\underline{w}}d^3$, causes the laws of $\delta\mathcal{E}_m$ particle physics to be further modified consistent with $\underline{\underline{w}}d^{1-2}$.

## 2. Derivation of $\underline{\underline{w}}d^3$

From the perspective of $`\underline{\Sigma}$, let a stationary *downstream clock*, $C_d$, and a stationary *upstream clock*, $C_u$, be separated by a vector-distance, $d`\underline{\mathbf{r}}$, that arbitrarily points from $C_u$ to $C_d$ in the $`\underline{\mathbf{p}}_m$ field, $`\underline{\mathbf{u}}$, making $`\underline{\mathbf{u}}\bullet d`\underline{\mathbf{r}} > 0$. And let electromagnetic radiation furnish the synchronization signals which travel in the opposite directions $`\underline{\mathbf{n}}^o_d$ and $`\underline{\mathbf{n}}^o_u = -`\underline{\mathbf{n}}^o_d$. Since $C_d$ will be reset first, the time of $C_d$ will be advanced relative to that of $C_u$ by an amount, $\delta`\underline{x}^0$, which is calculated via (3.22a) as

$$\delta`\underline{x}^0 \equiv [`\underline{x}^0(C_d) - `\underline{x}^0(C_u)] = \tfrac{1}{2}\,[d`\underline{r}\,/`\underline{\beta}^o_u - d`\underline{r}\,/`\underline{\beta}^o_d\,], \tag{3.28a}$$

$$= `\underline{\Gamma}^2\,(`\underline{\mathbf{u}}\bullet`\underline{\mathbf{n}}^o_d)d`\underline{r} \;= `\underline{\Gamma}^2(`\underline{\mathbf{u}}\bullet d`\underline{\mathbf{r}}) \equiv \underline{\underline{w}}d^3. \tag{3.28b}$$

From the perspective of $`\underline{\Sigma}$ then, $\underline{\underline{w}}d^3$ is a difference in time (or asimultaneity) that gets mapped to two neighboring clocks—independently of $\underline{\underline{w}}d^{1-2}$—via a GSS in $\underline{\Sigma}$ employing electromagnetic radiation as the sync signal. But the Null-$\underline{\mathbf{p}}_m$ Axiom could be violated, in principle, if a different value or function for $\delta`\underline{x}^0$ were to result from the use of any other kind of $\delta\mathcal{E}_m$ particle or field as the sync signal in a GSS. Hence it follows that (3.28b) must be independent of which $\delta\mathcal{E}_m$ particles or fields are employed as the synchronization signals in a GSS.

Conversely, one can gain a lot of information about the way that a given type of $\delta\mathcal{E}$–mass-energy flows—in the presence of a $`\underline{\mathbf{p}}_m$ field—by insisting that it flows in a way that would yield $\underline{\underline{w}}d^3$ if it were employed as the sync signal in a GSS. As the consequences of $\underline{\underline{w}}d^3$ (addressed next) will demonstrate, this is equivalent to simply insisting that all types of $\delta\mathcal{E}_m$–mass-energy flow in a Lorentz covariant manner.

## 3. Consequences of $\underline{\underline{w}}d^3$

From the perspective of $`\underline{\Sigma}$, it follows that the GRS defined transport time, $d`\underline{x}^0 = d`\underline{r}/`\underline{\beta}$, of a given $\delta\mathcal{E}_m$ particle in $`\underline{\Sigma}$ must be augmented by the corresponding asimultaneity, $\delta`\underline{x}^0$, in order to express the *total transport time*

$$D(`\underline{x}^0) \equiv (d`\underline{x}^0 + \delta`\underline{x}^0), \tag{3.29}$$

that two stationary, synchronized, clocks in $\underline{\Sigma}$ would measure, in compliance with $\underline{\underline{w}}d^2$, as the smaller time interval,

$$d\underline{x}^0 = D(`\underline{x}^0)/`\underline{\Gamma}, \tag{3.30}$$

for the GRS defined transport time, $d\underline{x}^0 = d\underline{r}/\underline{\beta}$, of that $\delta\mathcal{E}_m$ particle in $\underline{\Sigma}$.

The interpretation of $\delta`\underline{x}^0$; as a differential amount of asimultaneity that has been physically transferred to, or mapped to the clocks in $\underline{\Sigma}$, would be secured if it could be proved that (3.30) is correct. And because the laws of $\delta\mathcal{E}_m$ particle physics are known to be Lorentz covariant, we can do just that by proving that (3.30) is nothing less than the temporal component of a general Lorentz transformation—so that, taken together, $\underline{\underline{w}}d^{1-3}$ may be truthfully said to cause the $d\underline{x}^i$ of $\underline{\Sigma}$ to be related to the $dx^i$ of $\Sigma$ by a general Lorentz transformation.

*a. Proof that (3.30) is the temporal component of a general Lorentz transformation.*

Employing the Galilean relations of (2.4) and the previously defined equalities; $\underline{\underline{\beta}} = -\grave{\underline{\mathbf{u}}}$, and $\underline{\underline{\gamma}} = \grave{\underline{\Gamma}}$, allows us to express $\delta\grave{\underline{x}}^0$ in the following equivalent ways:

$$\delta\grave{\underline{x}}^0 = \grave{\underline{\Gamma}}^2(\grave{\underline{\mathbf{u}}}\bullet d\grave{\underline{\mathbf{r}}}) = \grave{\underline{\Gamma}}^2(\grave{\underline{\mathbf{u}}}\bullet\grave{\underline{\beta}})d\grave{\underline{x}}^0, \tag{3.31a}$$

$$= \grave{\underline{\Gamma}}^2(1 + \grave{\underline{\mathbf{u}}}\bullet\beta)d\grave{\underline{x}}^0 - d\grave{\underline{x}}^0, \tag{3.31b}$$

$$= \underline{\underline{\gamma}}^2(dx^0 - \underline{\underline{\beta}}\bullet d\mathbf{r}\ ) - d\grave{\underline{x}}^0, \tag{3.31c}$$

$$= \qquad D(\grave{\underline{x}}^0) \qquad - d\grave{\underline{x}}^0. \tag{3.31d}$$

And this permits $D(\grave{\underline{x}}^0)$ and $d\underline{x}^0$ to be expressed in terms of $\Sigma$ parameters, as

$$D(\grave{\underline{x}}^0) = \underline{\underline{\gamma}}^2(dx^0 - \underline{\underline{\beta}}\bullet d\mathbf{r}), \tag{3.32}$$

$$d\underline{x}^0 = \underline{\underline{\gamma}}(dx^0 - \underline{\underline{\beta}}\bullet d\mathbf{r}). \tag{3.33}$$

Thereby revealing that (3.30) is in fact the temporal component of a general Lorentz transformation,[20] and that the interpretation of $\delta\grave{\underline{x}}^0$; as a real asimultaneity that is generally manifested in two clock time measurements, is correct.

Since it has already been noted that the representation

$$d\underline{\mathbf{r}} = d\underline{\mathbf{r}}_{\parallel} + d\underline{\mathbf{r}}_{\perp} = \underline{\underline{\gamma}}(d\mathbf{r}_{\parallel} - d\underline{\underline{\mathbf{r}}}) + d\mathbf{r}_{\perp}, \tag{3.17f}$$

of $\underline{\underline{w}}d^1$ is the spatial component of a general Lorentz transformation, the above result also reveals that, taken together, $\underline{\underline{w}}d^{1-3}$ serve to predict that the $d\underline{x}^i$ of $\underline{\Sigma}$ are related to the $dx^i$ of $\Sigma$ by a general Lorentz transformation. This result, the catalytic utility of the $\grave{\underline{\Sigma}}$ perspective, and the Galilean relations of (2.4), are now exploited to derive relationships between the 3-scalar and 3-vector quantities; $\underline{\gamma}$, $\underline{\beta}$, $\gamma$, $\beta$, $\underline{\underline{\gamma}}$, and $\grave{\underline{\beta}}$, which are gainfully employed to derive the remaining $\underline{\underline{w}}d^{4-5}$ components of **w**-gauge theory.

4. Useful expressions of $\underline{\gamma}$ and $\underline{\beta}$ in terms of $\grave{\underline{\Sigma}}$ and $\Sigma$ parameters.

Dividing (3.17f) by (3.33) allows $\underline{\beta}$ to be expressed in terms of $\Sigma$ parameters as

$$\underline{\beta} = d\underline{\mathbf{r}}/d\underline{x}^0 = [\underline{\beta}_{\parallel} + \underline{\beta}_{\perp}] = [(\beta_{\parallel} - \underline{\underline{\beta}}) + \beta_{\perp}/\underline{\underline{\gamma}}\ ]/(1 - \underline{\underline{\beta}}\bullet\beta), \tag{3.34a}$$

and in terms of $\grave{\underline{\Sigma}}$ parameters as

$$\underline{\beta} \ = \underline{\underline{\gamma}}^2[\grave{\underline{\beta}}_{\parallel} + \grave{\underline{\beta}}_{\perp}/\underline{\underline{\gamma}}]/(1 - \underline{\underline{\gamma}}^2\underline{\underline{\beta}}\bullet\grave{\underline{\beta}}), \tag{3.34b}$$

$$= \underline{\underline{\gamma}}^2[\grave{\underline{\beta}}_{\parallel} + \grave{\underline{\beta}}_{\perp}/\underline{\underline{\gamma}}]/(1 + \delta\grave{\underline{x}}^0/dx^0). \tag{3.34c}$$

The parallel component of $\underline{\beta}$ covers what is popularly referred to as the *relativistic velocity addition* formula, with the new insight that $\delta\grave{\underline{x}}^0$ is directly responsible for $\underline{\underline{\beta}}\bullet\beta$ coupling term in the denominator of the formula, which is crucial for transforming $\underline{\beta}$ into $\underline{\beta}^0$ when $\beta \rightarrow \beta^0$ for electromagnetic radiation.[a]

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[a]From the perspective of $\Sigma$, if $\beta$ and $\underline{\underline{\beta}}$ are two known constants, then from the perspective of $\Sigma$ the GRV of a $\delta\mathcal{E}_m$ particle *relative to* $\underline{\Sigma}$ is correctly *predicted* in $\Sigma$ by $\grave{\underline{\beta}} = \beta - \underline{\underline{\beta}}$,.for all possible values of $\beta$ and $\underline{\underline{\beta}}$. This *velocity addition law* is immutable, which is why it and the other Galilean relations of (2.4) are so gainfully employed in descriptions referenced to $\grave{\underline{\Sigma}}$.

From (3.34a) one can readily obtain

$$\underline{\gamma} = \underline{\underline{\gamma}}\gamma(1 - \underline{\underline{\beta}}\bullet\beta) \tag{3.34d}$$

as the third member of the $\underline{\beta}(\beta, \underline{\underline{\beta}}, `\underline{\beta})$ GRV transformation group, with the $\underline{\underline{\beta}}\bullet\beta$ coupling term being tied directly to the asimultaneity described by $\delta`\underline{x}^0$.

*a. The inverse $\beta(\underline{\beta}, \underline{\underline{\beta}},)$ GRV transformation group.*

The parallel component of (3.34b) can be mathematically inverted to obtain $`\underline{\beta}_{\|}(\underline{\beta}_{\|}, \underline{\underline{\beta}})$ and thus $\beta_{\|}(\underline{\beta}_{\|}, \underline{\underline{\beta}})$ by making use of the formulas $`\underline{\beta}_{\|}\times\underline{\beta}_{\|} = \mathbf{0}$, and $\underline{\underline{\beta}}\bullet\mathbf{Z} \equiv \underline{\underline{\beta}}\bullet\mathbf{Z}_{\|}$. And the perpendicular component can then be mathematically inverted by making use of $\beta_{\|}(\underline{\beta}_{\|}, \underline{\underline{\beta}})$ and $`\underline{\beta}_{\perp} = \beta_{\perp}$. The resulting equations are

$$`\underline{\beta} = \beta - \underline{\underline{\beta}} = \frac{[\,\underline{\beta}_{\|}/\underline{\underline{\gamma}} + \underline{\beta}_{\perp}\,]}{\underline{\underline{\gamma}}\,(1 + \underline{\underline{\beta}}\bullet\underline{\beta})}, \tag{3.35a}$$

and

$$\beta = [(\underline{\beta}_{\|} + \underline{\underline{\beta}}) + \underline{\beta}_{\perp}/\underline{\underline{\gamma}}\,]/(1 + \underline{\underline{\beta}}\bullet\underline{\beta}), \tag{3.35b}$$

plus

$$\gamma = \underline{\underline{\gamma}}\,\underline{\gamma}\,(1 + \underline{\underline{\beta}}\bullet\underline{\beta}), \tag{3.35c}$$

as the third member of the $\beta(\underline{\beta}, \underline{\underline{\beta}},)$ GRV transformation group. Notice the symmetry of form that exists between $\beta(\underline{\beta}, +\underline{\underline{\beta}})$ and $\underline{\beta}(\beta, -\underline{\underline{\beta}})$, and between $\gamma(\underline{\beta}, +\underline{\underline{\beta}})$ and $\underline{\gamma}(\beta, -\underline{\underline{\beta}})$.

*b. Related formulas*

From (3.34d) and (3.335c) we obtain the related formulas,

$$\underline{\underline{\gamma}}^2(1 + \underline{\underline{\beta}}\bullet\underline{\beta})(1 - \underline{\underline{\beta}}\bullet\beta) = 1, \tag{3.36a}$$

$$(1 + \underline{\underline{\beta}}\bullet\underline{\beta})(1 - \underline{\underline{\gamma}}^2\,\underline{\underline{\beta}}\bullet`\underline{\beta}) = 1, \tag{3.36b}$$

$$\underline{\underline{\gamma}}^2(1 - \underline{\underline{\beta}}\bullet\beta) = (1 - \underline{\underline{\gamma}}^2\,\underline{\underline{\beta}}\bullet`\underline{\beta}). \tag{3.36c}$$

All of the reference frame transformations for the scalar and vector velocity parameters are thus seen to contain a second order coupling term between $\underline{\underline{\beta}}$ and the $\delta\mathcal{E}_m$ particle velocity which is tied directly to the temporal asimultaneity described by $\delta`\underline{x}^0$. These equations (3.34a–3.36c) are generally useful for investigating particle interactions that will preserve the Null-$\underline{\mathbf{p}}_m$ Axiom from the perspective of $`\underline{\Sigma}$, and this is illustrated herein by employing some of these equations to derive $\underline{\underline{w}}d^{4-5}$.

5. On the fundamental character of $\delta`\underline{x}^0$.

From the method of its derivation, one might be tempted to conclude that the temporal asimultaneity described by $\delta`\underline{x}^0$ only enters into the equations of $\delta\mathcal{E}_m$ particle physics whenever two clocks are employed to determine the time between two spatially separated events. But that would be an incorrect conclusion to draw, and being incorrect it would tend to blind one to the more fundamental character of the asimultaneity. We therefore summarize the results of two hypothetical experimental situations which suggest that the asimultaneity described by $\delta`\underline{x}^0$ has a more fundamental character than its "creation" via a GSS might lead one to believe. Thereby making room for the more general notion that the asimultaneity described by $\delta`\underline{x}^0$ is indigenous to the $`\underline{\mathbf{p}}_m$ field, and a GSS merely causes $\delta`\underline{x}^0$ to be accurately (Lorentz covariantly) mapped to the pair of clocks involved.

*a. The single-clock measure of $\underline{\beta}_{\|}$ via radar principles.*

In its simplest application, radar technology employs a single clock to measure the relative velocity of a target along a given *line of sight* (LOS), and since $\delta`\underline{x}^0$ is proportional to $d`\underline{\mathbf{r}}_{\|}$ there is no loss of substance if the outcome of such a measurement is modeled for the case that the direction of $\underline{\underline{\mathbf{r}}}$ is fixed and the GRS's $\beta$, $`\underline{\beta}$, and $\underline{\beta}$, of the target ($C^{\pi}$) are parallel to $\underline{\underline{\mathbf{r}}}$ and thus $\underline{\underline{\beta}}$. With respect to $\Sigma$, $\beta = \beta_{\|}$, is then determined via the dimensionless formula

$$\beta/(\beta^{o}_{A}\beta^{o}_{R}) = \beta/(1 - \beta) = \tfrac{1}{2}\Delta^2(x^0)/\Delta x^{o}_{p} = k. \tag{3.37a}$$

Here, $\beta^{o}_{A}$ is the speed with which electromagnetic radiation *approaches* $C^{\pi}$, $\beta^{o}_{R}$ is the speed with which electromagnetic radiation *returns* from $C^{\pi}$, $\Delta x^{o}_{p}$ is the time between successive outgoing radar pulses, and $\Delta^2(x^0)$ is the difference between two successive (round trip) echo times. Observers in $\underline{\Sigma}$ would naturally derive the same formula

$$\underline{\beta}/(\underline{\beta}^{o}_{A}\underline{\beta}^{o}_{R}) = \underline{\beta}/(1 - \underline{\beta}) = \tfrac{1}{2}\Delta^2(\underline{x}^0)/\Delta\underline{x}^{o}_{p} = \underline{k}, \tag{3.37b}$$

with $\underline{\beta} = \underline{\beta}_{\|}$. But from the theoretical perspective of $`\underline{\Sigma}$, employing (3.22a) to calculate the echo times, one obtains the slightly different formula

$$`\underline{\beta}/(`\underline{\beta}^{o}_{A}`\underline{\beta}^{o}_{R}) = `\underline{\beta}/[(1 - \beta)(1 + \underline{\underline{\beta}})] = \tfrac{1}{2}\Delta^2(`\underline{x}^0)/\Delta(`\underline{x}^{o}_{p}) = `\underline{k} = \underline{k}, \tag{3.37c}$$

where $`\underline{\beta} = `\underline{\beta}_{\|}$, $\beta = \beta_{\|}$, and the equality of $\underline{k}$ and $`\underline{k}$ follows from the $\underline{\underline{w}}d^2$ relations; $\Delta^2(`\underline{x}^0) = `\underline{\Gamma}\Delta^2(\underline{x}^0)$, and $\Delta`\underline{x}^{o}_{p} = `\underline{\Gamma}\,\Delta\underline{x}^{o}_{p}$.

The empirical isotropy of $\underline{\Sigma}$ (necessitated by the Null-$\underline{\mathbf{p}}_m$ Axiom) will then be preserved regardless of the $`\underline{\mathbf{u}}$ bias of electromagnetic radiation provided that the laws of $\delta\mathcal{E}_m$ particle physics (including $\underline{\underline{w}}d^2$) uphold the equality

$$\underline{\beta}/(1 - \underline{\underline{\beta}}) = `\underline{\beta}/[(1 - \beta)(1 + \underline{\underline{\beta}})], \tag{3.38a}$$

$$= `\underline{\beta}/(1 - `\underline{\beta} - \underline{\underline{\beta}}\beta). \tag{3.38b}$$

Which is to say, provided that

$$\underline{\beta} = (\beta - \underline{\underline{\beta}})/(1 - \underline{\underline{\beta}}\beta). \tag{3.39}$$

Here it is made clear that the $\underline{\underline{\beta}}\beta$ coupling term comes into play from $\underline{\beta}^{o}_{R}.= (1 + \underline{\underline{\beta}}).= (1 + `\underline{u})$.

*b. Derivation of $\underline{\underline{w}}d^3$ from $\underline{\underline{w}}d^1$ and the reference frame invariance of $\underline{\underline{w}}d^2$.*

Given $\underline{\underline{w}}d^{1–2,}$ Eqs. (2.4), and the equalities $\ell = `\underline{\ell} = \gamma$ noted in (III.A.1.a), the propagationally reduced time increment registered by a clock must have, in $\Sigma$, $\underline{\Sigma}$, and $`\underline{\Sigma}$, the same value

$$dS = dx^0/\gamma = d`\underline{x}^0/\gamma = d\underline{x}^0/\underline{\gamma}, \tag{3.39b}$$

$$= dr/\gamma\beta = d`\underline{r}/\gamma(`\underline{\beta}) = d\underline{r}/\underline{\gamma}\underline{\beta} = d`\underline{r}[1 - (`\underline{\mathbf{n}}\times`\underline{\mathbf{u}})^2]/\underline{\gamma}\underline{\beta}. \tag{3.39b}$$

Using $(\gamma\beta)^2 = (\gamma^2 - 1)$, equations (3.39a-b) can be solved for $\underline{\gamma}$, giving

$$\underline{\gamma} = \gamma[1 + `\underline{\Gamma}^2(`\underline{\mathbf{u}}\bullet`\underline{\beta})]/`\underline{\Gamma} = `\underline{\Gamma}\gamma(1 + `\underline{\mathbf{u}}\bullet\beta) = \underline{\underline{\gamma}}\gamma(1 - \underline{\underline{\beta}}\bullet\beta). \tag{3.39c}$$

And by then using (3.39c) in (3.39a) to eliminate $\underline{\gamma}$, one obtains

$$d\underline{x}^0 = [d`\underline{x}^0 + `\underline{\Gamma}^2(`\underline{\mathbf{u}}\bullet d`\underline{\mathbf{r}})]/`\underline{\Gamma} \equiv (d`\underline{x}^0 + \delta`\underline{x}^0)/`\underline{\Gamma} \equiv D(`\underline{x}^0)/`\underline{\Gamma}. \tag{3.39d}$$

Q.E.D.

### J. A simple derivation of the **w**–deformation of $p^{\pi}$ particle-mass ≡ $\underline{\underline{w}}d^4$.

#### 1. The propagational dynamics of a freely spinning mass.

Let the state of $\underline{\Sigma}$ corresponding to $\grave{}\underline{\mathbf{u}} = \mathbf{0}$ be denoted by $\underline{\Sigma}^0$. We then define, in $\underline{\Sigma}^0$ and in $\grave{}\underline{\Sigma}^0$, the angular momentum $\mathbf{H}^0 = I^0\, \varpi^0$ of an *idealistic frictionless gyroscope* (IFG) of mass $m^0$ and moment of inertia $I^0 = km^0$ for a given constant k. The IFG is also a simplistic mechanical clock, with one countable rotation defining a time period of $T^0 = 2\pi/\varpi^0$ seconds.

We then consider, from the perspective of $\grave{}\underline{\Sigma}$, the state of the IFG after the appearance of a constant $\grave{}\underline{\mathbf{u}}$ field and conclude that the angular rotation rate of the IFG must decrease from $\grave{}\underline{\varpi} = \varpi^0$ to the $\grave{}\underline{\varpi} = \varpi^0/\grave{}\underline{\Gamma}$ in order to comply with $\underline{\underline{w}}d^2$.

But it is also necessary that the angular momentum of the IFG be conserved. Hence, from the perspective of $\grave{}\underline{\Sigma}$ it must be true that $\grave{}\underline{\mathbf{H}} = \mathbf{H}^0 = \grave{}\underline{I}\ \grave{}\underline{\varpi} = (k\grave{}\underline{m})\ \varpi^0/\grave{}\underline{\Gamma}$. Which is possible only if the mass of the IFG increases in accordance with the law $\grave{}\underline{m} = \grave{}\underline{\Gamma}m^0$. Hence $\underline{\underline{w}}d^2$, **H**-conservation, and $\grave{}\underline{m} = \grave{}\underline{\Gamma}m^0$ are interdependent, and since we have already shown $\underline{\underline{w}}d^2$ to be a dependent **w**-deformation it follows that, in this particular case, $\underline{\underline{w}}d^2$ is explained by **H** conservation and a propagationally induced increase of mass. The $\underline{\underline{w}}d^4$ component of **w-**gauge theory, arbitrarily referenced to Σ, is then formally described more generally by

$$\underline{\underline{w}}d^4 = m/m^0 = \ell = 1/[1 - \mathbf{w}^2]^{1/2}. \qquad (3.40)$$

#### 2. On the wholly dependent nature of $\delta\mathcal{E}_m$ time-dilation.

The preceding finding therefore strengthens the two conclusions drawn in III.H.1: That $\underline{\underline{w}}d^2$ is a totally dependent member of the group of $\underline{\underline{w}}d^i$ deformations. And, accordingly, logical place to seek a physical explanation of time is in the physics accounting for the structure of the ${}^0\!\in_m$–continuum.

### K. Derivation of $\underline{\underline{w}}d^4$ from an analysis of pairs of particles having zero net-momentum in $\underline{\Sigma}$.

We now derive $\underline{\underline{w}}d^4$ by employing $\grave{}\underline{\Sigma}$ to:

a) Analyze the results of an experiment in $\underline{\Sigma}$ designed to generate a pair of identical $\delta\mathcal{E}_m$ particles having equal and opposite coordinate velocities and momenta relative to $\underline{O}$.

b) Deduce that $\underline{\underline{w}}d^4$ is necessary to prevent the Null-$\underline{\mathbf{p}}_m$ Axiom from being violated by the observations of both unequal GRV's and unequal momenta.

The notations employed to describe the experiment—referred to as the ←(2)[($\underline{O}$)](1)→ experiment—are described next.

#### 1.Notations for a ←(2)[($\underline{O}$)](1)→ experiment in $\underline{\Sigma}$

We denote two identical $\delta\mathcal{E}_m$ particles as $\delta\mathcal{E}_m$ particles (1) and (2). And we define a ←(2)[($\underline{O}$)](1)→ Energy Conversion in $\underline{\Sigma}$ as one that is designed to impart equal and opposite GRV's and momenta to these particles—on the presumption that physical space of $\underline{\Sigma}$ is equivalent to the ${}^0\!\in_m$–continuum.

From the derivation of $\underline{\underline{w}}d^3$ we know that the influence of $\underline{\underline{w}}d^3$ vanishes for $\underline{\beta}$ perpendicular to $\grave{}\underline{\mathbf{u}}$. Hence we will analyze the ←(2)[($\underline{O}$)](1)→ experiment, without loss of substance, by taking the GRV of each $\delta\mathcal{E}_m$ particle be parallel or anti-parallel to $\grave{}\underline{\mathbf{u}}$, which allows us to work with the GRS of each $\delta\mathcal{E}_m$ particle instead.

Hence we expect to find that $\underline{\beta}_1 = \underline{\beta}_2 \equiv \underline{\beta}$ , and that $\underline{p}_1 = \underline{p}_2 \equiv \underline{p}$. But we will employ $\underline{\underline{w}}d^{1-3}$ and the GRV formula, to calculate $\underline{\beta}_x$ as a function of $`\underline{\beta}_x$ (x = 1 or 2), and thereby show that this symmetry can not exist unless $\underline{\underline{w}}d^4$ intervenes to cause $\delta\mathcal{E}_m$ energy and momentum to be conserved in compliance with the Null-$\underline{\mathbf{p}}_m$ Axiom.

2. The true asymmetry of $`\underline{\beta}_1$ and $`\underline{\beta}_2$ in $`\underline{\Sigma}$ and its consequences.

In order for the Null-$\underline{\mathbf{p}}_m$ Axiom to be preserved, it is clear that observers in $\underline{\Sigma}$ must find that $\underline{\beta}_1 = \underline{\beta}_2$ for any orientation ($\underline{\mathbf{n}}$) of the ←(2)[($\underline{O}$)](1)→ apparatus. Considering the left side of (3.35a), however, it is equally clear that $`\underline{\beta}_1$ and $`\underline{\beta}_2$ will then differ in some $`\underline{\mathbf{u}}$-dependent and $\underline{\beta}$–dependent way. Hence, in order to preserve the Null-$\underline{\mathbf{p}}_m$ Axiom we are obligated to complete the following two tasks:

a) Derive the values of $`\underline{\beta}_1$ and $`\underline{\beta}_2$ corresponding to $\underline{R} \equiv \underline{\beta}_1/\underline{\beta}_2 = 1$.

b) Derive a **w**–deformation which will explain why the values of $`\underline{\beta}_1$ and $`\underline{\beta}_2$ so determined are physically necessary.

*a. Task–a): The values of* $`\underline{\beta}_x$ *corresponding to* $\underline{\beta}_1 = \underline{\beta}_2$ .

Solving (3.40a) for $`\underline{\beta}$ gives

$$`\underline{\beta} = \frac{[\underline{\beta}^2 - (\underline{\beta}\bullet`\underline{\mathbf{u}})^2]^{1/2}}{`\underline{\Gamma}\,(1 - \underline{\beta}\bullet`\underline{\mathbf{u}}\,)} . \tag{3.43}$$

And since $(\underline{\beta}_1)^2 = (\underline{\beta}_2)^2$, it follows that

$$`\underline{R} \equiv \frac{`\underline{\beta}_1}{`\underline{\beta}_2} = \frac{(1 - \underline{\beta}_2\bullet`\underline{\mathbf{u}}\,)}{(1 - \underline{\beta}_1\bullet`\underline{\mathbf{u}}\,)} . \tag{3.44}$$

*b. Task–2): The necessary* ***w****–deformation,* $\underline{\underline{w}}d^4$.

In accordance with Newtonian mechanics, the asymmetry of $`\underline{\beta}_1$ and $`\underline{\beta}_2$ described by (3.44) could only exist if a momentum impulse of magnitude $\Delta`\underline{\mathbf{J}} = |`\underline{m}_2`\underline{\beta}_2 - `\underline{m}_1`\underline{\beta}_1|$ was imparted to the ←(2)[($\underline{O}$)](1)→ apparatus—which could be easily instrumented to sense it. Hence, the equality of $\underline{\beta}_1$ and $\underline{\beta}_2$, although *necessary,* is *insufficient* to preserve the conservation law of linear momentum for particle pairs generated by a symmetric ←(2)[($\underline{O}$)](1)→ energy conversion.

It follows that if the conservation law for the linear momentum of $p^\pi$ particles is to be preserved with momentum expressed in the Newtonian from, $`\underline{\mathbf{p}} = `\underline{m}`\underline{\beta}$, it is then necessary to conclude that the mass of a $p^\pi$ particle varies with its propagation velocity in such a way that $`\underline{m}_1/`\underline{m}_2 = m_1/m_2 = 1/`\underline{R}$. Using this restriction, (3.35c), and the equivalences described in III.A and III.F, we obtain

$$\frac{`\underline{m}_2}{`\underline{m}_1} = \frac{m_2}{m_1} = \frac{`\underline{\beta}_1}{`\underline{\beta}_2} = \frac{(1 - \underline{\beta}_2\bullet`\underline{\mathbf{u}})}{(1 - \underline{\beta}_1\bullet`\underline{\mathbf{u}})} = \frac{\ell_2}{\ell_1} = \frac{\gamma_2}{\gamma_1} = \frac{(1 - \beta_2^{\,2})^{1/2}}{(1 - \beta_1^{\,2})^{1/2}} = \frac{(1 - \mathbf{w}_2^{\,2})^{1/2}}{(1 - \mathbf{w}_1^{\,2})^{1/2}} , \tag{3.45}$$

and thus, the result

$$\underline{\underline{w}}d^4 = m/m^0 = \ell = 1/[1 - \mathbf{w}^2]^{1/2}, \tag{3.40}$$

previously derived in III.J for the **w**–deformation of $p^\pi$ particle mass referenced arbitrarily to Σ.

*c. The mass and velocity asymmetries of zero-momentum particle pairs.*

The asymmetries of the particle masses and velocities implied by (3.45) is more fully described and clarified by noting that the value of $\grave{\underline{\beta}}\bullet\grave{\underline{\mathbf{u}}}$ will be *positive* for the particle that is moving in a generally *downstream direction.* That particle is here defined as $p_2$ but re-indexed as $p_d$ for added clarity. Hence $(1 - \underline{\beta}_d\bullet\grave{\underline{\mathbf{u}}}) \rightarrow (1- |\underline{\beta}_d\bullet\grave{\underline{\mathbf{u}}}|)$. Conversely, for the *upstream particle* ($p_1 \rightarrow p_u$) we have $(1 - \underline{\beta}_u\bullet\grave{\underline{\mathbf{u}}}) \rightarrow (1+ |\underline{\beta}_u\bullet\grave{\underline{\mathbf{u}}}|)$. From these definitions and (3.45) we then obtain

$$\grave{\underline{R}} = \frac{\grave{\underline{m}}_d}{\grave{\underline{m}}_u} = \frac{\grave{\underline{\beta}}_u}{\grave{\underline{\beta}}_d} = \frac{(1 - |\underline{\beta}_d\bullet\grave{\underline{\mathbf{u}}}|)}{(1 + |\underline{\beta}_u\bullet\grave{\underline{\mathbf{u}}}|)} = \frac{\grave{\underline{\ell}}_d}{\grave{\underline{\ell}}_u} = \frac{(1-\grave{\underline{\mathbf{w}}}_u^2)^{1/2}}{(1-\grave{\underline{\mathbf{w}}}_d^2)^{1/2}} \leq 1. \tag{3.46a}$$

Hence, relative to $\grave{\underline{\Sigma}}$,

$$\grave{\underline{\beta}}_u < \grave{\underline{\beta}}_d, \tag{3.46b}$$

$$\grave{\underline{m}}_u = m_u > \grave{\underline{m}}_d = m_d, \text{ because } \grave{\underline{w}}_u = w_u > \grave{\underline{w}}_d = w_d. \tag{3.46c}$$

That is, compared to $p_d$,

> $p_u$ has a *larger* propagation speed ($\grave{\underline{w}}_u = w_u = \beta_u$)—accounting for a *larger* upstream mass ($\grave{\underline{m}}_u = m_u$) which is transported *upstream* with a *smaller* GRV speed ($\grave{\underline{\beta}}_u$) so that $m_u\grave{\underline{\beta}}_u = m_d\grave{\underline{\beta}}_d$.

And conversely, compared to $p_u$,

> $p_d$ has a *smaller* propagation speed ($\grave{\underline{w}}_d = w_d = \beta_d$) accounting for a *smaller* downstream mass ($\grave{\underline{m}}_d = m_d$) which is transported *downstream* with a *larger* GRV speed ($\grave{\underline{\beta}}_d$) so that $m_d\grave{\underline{\beta}}_d = m_u\grave{\underline{\beta}}_u$.

*d. Additional consequences of* $\grave{\underline{\mathbf{p}}}_1 + \grave{\underline{\mathbf{p}}}_2 = \mathbf{0}$

Given $\underline{\underline{w}}d^{1-4}$, setting $\grave{\underline{\mathbf{p}}}_1 + \grave{\underline{\mathbf{p}}}_2 = \mathbf{0}$, and using $\grave{\underline{\beta}}_x = (\beta_x - \underline{\beta})$ and $\ell_x = \underline{\ell}_x\,\underline{\underline{\ell}}(1 + \underline{\beta}_x\bullet\underline{\beta})$, we obtain the equations

$$\mathbf{p}_1 + \mathbf{p}_2 = m_1\beta_1 + m_2\beta_2 = (m_1 + m_2)\underline{\underline{\beta}} \equiv M\,\underline{\underline{\beta}} = \underline{M}\,\underline{\beta}, \tag{3.47a}$$

since
$$M = m_0(\ell_1 + \ell_2) = 2m_0\underline{\underline{\ell}}\,\underline{\ell} = 2\underline{m}_0\underline{\ell} = 2\underline{m} = \underline{M}, \tag{3.47b}$$

and,
$$\Delta M = M - 2m_0\underline{\underline{\ell}} = \Delta\underline{M} = \underline{M}_0\,(\underline{\ell} - 1), \tag{3.48a}$$

$$\cong \underline{M}_0\,(\underline{\mathbf{w}}^2/2) = \Delta\underline{KE}/c^2 \text{ for } \underline{\mathbf{w}}^2 << 1. \tag{3.48b}$$

If $\Delta\underline{\Phi}$ is the potential energy used up by the symmetric $\leftarrow(2)[(\underline{O})](1)\rightarrow$ energy conversion in $\underline{\Sigma}$, these equations show that $\Delta\underline{\Phi}$ has the following three characteristics:

1) $\Delta\underline{\Phi}$ creates an increase in the propagating masses of $p_1$ and $p_2$, $\Delta M = \Delta\underline{M}$, which is invariant under the $\mathbf{w}$−deformations $\underline{\underline{w}}d^{1-4}$ linking $\underline{\Sigma}$ to $\Sigma$, and which leaves the total propagating mass ($M = \underline{M}$) reference-frame invariant as well.

2) From the perspective of $\Sigma$, $\Delta\underline{\Phi}$ produces a non-zero change of momentum in $\Sigma$, $\Delta\mathbf{p} = \Delta m\underline{\beta}$, which is consistent with $\Delta\grave{\underline{\mathbf{J}}} = \Delta\underline{\mathbf{p}} = \mathbf{0}$ in $\underline{\Sigma}$.

3) For $\beta$ and $\underline{\beta} << 1$, the $\Delta M$ and $\Delta\underline{M}$ supplied by $\Delta\underline{\Phi}$ are both equal to $1/c^2$ times the changes in the corresponding Newtonian kinetic energies of the particles, $\Delta KE$ and $\Delta\underline{KE}$.

These characteristics of $\Delta\underline{\Phi}$ suggest that, in order for the conservation of mass and energy to be maintained, $\Delta\underline{\Phi}$ must possess $\Delta\underline{\Phi}/c^2$ *kilograms of mass* or $\Delta\underline{\Phi}$ Joules of *mass-energy*—at least in the domain of $\underline{\beta} << 1$. There are several ways to demonstrate that the Null-$\underline{\mathbf{p}}_m$ Axiom would be violated, however, if this $c^2$ link between energy and mass failed to hold good for $\underline{\beta} \to 1$, and the following is one example

L. $\underline{\underline{w}}d^5$: The physical equivalence of a $\delta\mathcal{E}_m$ particle's mass and energy.

In the derivation of $\underline{\underline{w}}d^4$ we employed the Newtonian expression for $p^\pi$ particle momentum ($\mathbf{p} = mc\beta$) to show that, in order for mass and momentum to be **w-**gauge invariant, mass must be expressed in the from $m = m_0\gamma$. If we further assume that Newton's equation for a differential change of a $p^\pi$ particle's kinetic energy ($d(KE) = \mathbf{F}\bullet d\mathbf{r}$) also remains **w**-gauge invariant, with $\mathbf{F} = cd\mathbf{p}/dx^0$ being Newton's formula for the force acting on the particle, we then get the following differential equation

$$d(KE) = \mathbf{F}\bullet d\mathbf{r} = c\beta\bullet d\mathbf{p} = \frac{1}{2m}d(\mathbf{p}^2) = \frac{m_0c^2}{2\gamma}d(\gamma\beta)^2 = d(mc^2) \tag{3.49}$$

with the indefinite solution

$$KE = mc^2 + C. \tag{3.50}$$

Using the constraint [$KE(\beta = 0) = 0$] to solve for the constant C yields an equation

$$KE = mc^2 - m_0c^2 = \Delta mc^2 = m_0c^2(\ell - 1) = E^0(\ell - 1) = E - E^0, \tag{3.51a}$$

$$\cong \frac{1}{2}m_0v^2 \quad \text{for } \mathbf{w}^2 = \beta^2 << 1, \tag{3.51b}$$

for the propagational energy of a $p^\pi$ particle, where $E^0 = m_0c^2$ is an energy associated with the particle's structure when it is at rest in the ${}^0\in_m$ continuum, and $E = mc^2$ is clearly the particle's total (propagational plus non propagational) energy. In this way we have proved the prior suspicion; that the $c^2$ link between particle mass and energy holds good for $w \to 1$, and via (3.51b) we see that the Newtonian expression for a $p^\pi$ particle's kinetic energy naturally obtains when the particle's propagation speed is small compared to lightspeed.

1. The resulting physical equivalence of $p^\pi$ particle mass and energy, and thus, of $\underline{\underline{w}}d^4$ and $\underline{\underline{w}}d^5$.

Solving Eq. (3.51a) for $mc^2$ puts the foregoing results in a clearer perspective by showing that a $p^\pi$ particle evidently carries a total energy

$$E(\ell) \equiv m(\ell)c^2 = [\, KE(\ell) + E^0 \,] = E^0\ell \tag{3.52a}$$

which is defined in three equivalent ways by:

$$E(\ell) = [\, KE(\ell) + E^0 \,], \tag{3.52b}$$

$$E(\ell) = E^0\ell, \tag{3.52c}$$

$$E(\ell) = m(\ell)c^2. \tag{3.52d}$$

The $\underline{\underline{w}}d^5$ component of **w-**gauge theory is then obtained explicitly from Eq. (3.52a) as

$$\underline{\underline{w}}d^5 \equiv E/E^0 = m/m_0 = \ell. \tag{3.53}$$

This shows that $\underline{\underline{w}}d^4$ and $\underline{\underline{w}}d^5$ are physically equivalent because the energy and inertia of a $p^\pi$ particle are physically equivalent. Which strongly suggests that a $p^\pi$ particle's *inertia* and *energy* are just *different aspects* of ${}^0\in_m$–$p^\pi$ *coupling physics*—and this is indeed qualitatively explained by the compactification model described in Sections VII – IX.

M. The complete $\underline{\underline{w}}d^i$ group.

The complete $\underline{\underline{w}}d^i$ group of **w**–deformations is then

$$\underline{\underline{w}}d^1 = \underline{\underline{w}}d^2 = \underline{\underline{w}}d^4 = \underline{\underline{w}}d^5 = \ell$$
$$\frac{V^0}{V_\|} = \frac{dt}{dt^0} = \frac{m}{m^0} = \frac{E}{E^0} = \ell \equiv 1/[1-\mathbf{w}^2]^{1/2}, \tag{3.54}$$

plus the differential asimultaneity component,

$$\underline{\underline{w}}d^3 = \delta\grave{\underline{x}}^0 = \grave{\underline{\Gamma}}^2(\grave{\underline{\mathbf{u}}}\bullet d\grave{\underline{\mathbf{r}}}) = -\underline{\underline{\ell}}^2(\underline{\underline{\mathbf{w}}}\bullet d\grave{\underline{\mathbf{r}}}) = -\underline{\underline{\gamma}}^2(\underline{\underline{\beta}}\bullet d\grave{\underline{\mathbf{r}}}) \text{ light-seconds}, \tag{3.32d}$$

and the implicit equivalence of $\delta\mathcal{E}_m$ mass and energy

$$E(\ell) = m(\ell)c^2. \tag{3.52d}$$

All of which are:

- Conjointly necessary to preserve the Null-$\underline{\mathbf{p}}_m$ Axiom.
- Implicitly contained in the (1 + 3) tensor structure of EM gravity, which covers special relativity theory for the restriction $\mathbf{u}(x^i) = \mathbf{0}$ (to the γ-coupling domain).
- Vital for an intuitive hyper-energy understanding of the tensor mathematics of EM gravity which readily precipitates all of the $\underline{\underline{w}}d^i$—but with no hint of their relative priorities.
- Necessary, therefore, to prevent their traditional (null ${}^0\in_m$) interpretations from weakening the reader's comprehension of EM gravity.

1. Can the $\underline{\underline{w}}d^i$, the ${}^0\in_m$-continuum, the roots of EM gravity, and the necessity of a (1 + *p*)-dimensional energy reality and universe theory, all be derived from $E = mc^2$?

Because it was just shown that the equality of $\underline{\underline{w}}d^4$ and $\underline{\underline{w}}d^5$ is intrinsically linked to equality of particle energy and mass described by $E = mc^2$, one can readily guess that the answer to the title question is yes. The details of just such a derivation are in fact provided in my earlier work[3].

N. Experimental confirmation of Maxwell's gravity theorem and Maxwellian gravity.

1. Introduction.

Via **w-**gauge theory we have just shown that the Maxwellian gauge field, $\mathbf{p}_m(x^i) = \in_m(x^i)\mathbf{u}(x^i)$, preserves the (1 + 3) symmetry required by special relativity theory, but for the revolutionary physical condition, ${}^0\in_m >> 0$, instead of the physical condition ${}^0\in_m = 0$ that was thought to be uniquely dictated by relativity theory. But if $\mathbf{p}_m(x^i)$ is that fundamental, any temporal or spatial variation of $\mathbf{p}_m$ should create a corresponding $\in_m$-field that can be readily observed. So before introducing the tensor formalism of Einstein-Maxwell gravity, it is important to show that—within the limits of the 3-vector calculus—which characterizes Maxwellian gravity—three such $\in_m$-fields can now be seen to explain and to unify:

- The non conservative spatial field of 'merry-go-round' gravity, as; $\underline{\mathbf{g}}_r \equiv \underline{\boldsymbol{\zeta}} = (\underline{\nabla}\times\underline{\mathbf{u}}) \cong \grave{\boldsymbol{\zeta}}$.
- The non conservative temporal field of 'elevator' gravity, as; $\underline{\mathbf{g}}_0 \equiv \underline{\mathbf{u}}_0 = \partial\underline{\mathbf{u}}/\partial\underline{x}^0 \cong \grave{\underline{\mathbf{u}}}_0$.
- The conservative spatial field of Newtonian gravity, as; $\underline{\mathbf{g}}_s \equiv \underline{\nabla}(\tfrac{1}{2}\underline{\mathbf{u}}^2) \equiv \underline{\nabla}(\underline{\varphi}_m) \cong.\grave{\underline{\mathbf{g}}}_s$.
- The workings, therefore, of the various *gravity* and *inertial acceleration* sensors which are ubiquitously employed today by *inertial navigation systems* to control the orientations ($\theta_i$) and propaga-

tion paths ($S_i$) of spacecraft throughout the ${}^0\in_m$–continuum of so-called *inertial space* that subsists after the distortions of Newtonian gravity along each $S_i$ have been mathematically removed.

With all of this substantiated, the reader can be confident that Maxwellian gravity offers a fundamental, albeit mathematically limited, 3-vector resolution of the quantum gravity problem. The reader can then proceed to EM gravity and the GFT3 with a much greater confidence in, and understanding of, the new hyper-energy physics which is further evolved therein. Additionally, it will be made clear that inertial navigation theorists can immediately begin to model *inertial sensor response functions*—more conveniently and consistently than before—from a proper (in situ) field theoretic perspective of the specific $\in_m$-fields that are actually being sensed.

*a. Outline of experimental evidence for the* $\underline{\boldsymbol{\zeta}}$*-field.*

We will first employ the GRV equation of electromagnetic radiation to calculate the proper electromagnetic frequency response $\Delta\underline{\mathbf{f}}(\underline{\boldsymbol{\zeta}})$ of a ring laser to a local $\underline{\boldsymbol{\zeta}}$-field. And we will then employ the relative GAV equation to point out the special conditions under which $\Delta\underline{\mathbf{f}}(\underline{\boldsymbol{\zeta}})$ can be accurately correlated to the ring-laser's $\underline{\underline{\omega}}$, and thus, the special conditions—which are not presently acknowledged and appreciated—under which a ring laser can be also regarded as a *rotation sensor*.

*b. Outline of experimental evidence for the Maxwellian gravity field* $\mathbf{g}_m = \mathbf{g}_0 + \mathbf{g}_s$.

We next employ the relative GRA equation to show that $\underline{\mathbf{g}}_0$ provides a fundamental explanation for the field of 'elevator' gravity—as measured by a *restrained mass* (RM) instrument, functioning as $\underline{\mathbf{g}}_0$-field sensor—and we then point out the special conditions under which $\underline{\mathbf{g}}_0$ can be accurately correlated to the RM's $\underline{\underline{\beta}}_0$, and thus, the special conditions—which are not presently acknowledged and appreciated—under which an RM instrument can also be pragmatically regarded as an *inertial acceleration* sensor or *accelerometer*.

Then, recalling the fact that 'elevator' gravity and Newtonian gravity are known to be just different aspects of the same*, unknown,* physical thing, we note that $\mathbf{g}_s$ is both mathematically identical to the field of Newtonian gravity, and physically equivalent to $\underline{\mathbf{g}}_0$—since $\mathbf{g}_0$ and $\mathbf{g}_s$ are just the different *temporal* and *conservative-spatial* aspects of the 3-vector, $d\mathbf{u}/dx^0$, describing the *total acceleration* of $\in_m$.

In this way it is shown that the Maxwellian gravity field, $\mathbf{g}_m = \mathbf{g}_0 + \mathbf{g}_s$ provides a unified explanation of 'elevator' gravity and Newtonian gravity, and that, consequently, the potential, $\Phi_m \equiv \frac{1}{2}\mathbf{u}^2$, of *ideal Maxwellian gravity* provides an explanation of the absolute potential, $-\Phi_n/c^2$, of *ideal Newtonian gravity,* commensurate with limitations of the 3-vector subsection of the (1 + 3) sector of hyper-energy theory.

*c. Outline of evidence that Einstein gravity implicitly contains Maxwellian Gravity.*

The revelation, $\Phi_m = \frac{1}{2}\mathbf{u}^2 = -\Phi_n/c^2$, of Maxwellian gravity, implies that ideal Newtonian gravity is the first *matter field* to be fundamentally addressed, even if incompletely, within the 3–vector subsection of the (1 + 3) sector of hyper-energy theory. The consequences of EM gravity will transform this implication to a certainty, but in keeping with the intent of this subsection; to describe the experimental evidence for Maxwellian gravity—as a insightful prelude to EM gravity—this subsection is concluded by first noting that: a) the $\Phi_m$ explanation of $-\Phi_n/c^2$ can be expressed in the form $[1 + 2\Phi_n/c^2] = [1 - \mathbf{u}^2] = 1/\Gamma^2$, and b) this can then be used to transform the spherically symmetric, general relativistic, metric interval for ideal Newtonian gravity into an equivalent hyper-energy expression, $ds^2 = \{ (dx^0/\Gamma)^2 - (\Gamma dr)^2 - r^2[d\theta^2 + \sin^2\theta d\varphi^2] \}$, as one example of how the $\in_m$–particle coupling of **w-**gauge theory will naturally surface within EM gravity.

It is then argued that the gravitational red-shift, resulting as it does from the gravitational reduction of any oscillator's frequency, can be intuitively understood now in terms of the $\in_m$–particle coupling that obtains in the Γ-coupling domain of the general $\in_m$–particle coupling. Also, since ds = 0 characterizes electromagnetic radiation, this predicts that, in the field of ideal Newtonian gravity, electromagnetic radiation propagates radially with a speed $dr/dx^0 = (1 - \mathbf{u}^2)$. And we then explain how this invites an interpretation of a black hole as a stationary $\in_m$–shockwave.

Finally, we note that the interpretation of a black hole as an $\in_m$-shockwave correctly suggests a general principle of hyper-energy theory which is richly exploited in GFT3, and is in harmony with the ongoing efforts to extract *hints of the new physics* from thermodynamic analyses of *black-hole physics*. Namely; that $\in_m$–*compressibility* is the root cause of the *non-linearity* which gives the hyper-energy field equations their enormous *structure-forming potential*—permitting *apparent* singularities associated with the compactified 3–space-energy continuum, the structures of electrically charged and neutral $\delta\mathcal{E}_m$ field-particles, and the *apparent* lightspeed limitation to $\delta\mathcal{E}_m$ propagation, to be seen as just that, *apparent* singularities and limitations.

## 2. The fields, $\underline{\mathbf{g}}_r$, $\underline{\mathbf{g}}_0$, and $\underline{\mathbf{g}}_s$, of Maxwellian gravity.

Relative to the $\underline{\Sigma}$ of a spacecraft (named $\underline{\Sigma}$), first order temporal and spatial variations of $\underline{\mathbf{p}}_m(\underline{x}^i)$ generate three observable flow fields of $\in_m \leq {}^0\in_m$ denoted (with italics) by $\underline{\boldsymbol{g}}_r$, $\underline{\boldsymbol{g}}_0$, and $\underline{\boldsymbol{g}}_s$. Letting $k = c/\in_m$, these three $\in_m$-fields describe:

- The $\in_m$-circulation field of $\in_m$*-vorticity*, $\underline{\boldsymbol{g}}_r \equiv \underline{\boldsymbol{\zeta}} = k\,(\underline{\nabla}\times\underline{\mathbf{p}}_m) \equiv 2\,\underline{\boldsymbol{s}}$, (3.55a)
- The $\in_m$-acceleration field of *temporal gravity*', $\underline{\boldsymbol{g}}_0 \equiv ck\,\partial\underline{\mathbf{p}}_m/\partial\underline{x}^0$, (3.56a)
- The $\in_m$-acceleration field of *spatial gravity*, $\underline{\boldsymbol{g}}_s = k^2\,\underline{\nabla}(\tfrac{1}{2}\underline{\mathbf{p}}_m^2) \equiv \underline{\nabla}(\underline{\varphi}_m)$, (3.57a)

### *a. Some convenient working approximations and notations.*

The $\underline{\boldsymbol{g}}_0$, $\underline{\boldsymbol{g}}_s$, and $\underline{\boldsymbol{g}}_r$ fields of Maxwellian gravity defined via Eqs. (3.55a–3.57a) are conveniently referred to as the $\underline{\boldsymbol{G}}_m$ fields of Maxwellian gravity, with $\underline{\boldsymbol{G}}_1 \equiv \underline{\boldsymbol{g}}_0$, $\underline{\boldsymbol{G}}_2 \equiv \underline{\boldsymbol{g}}_s$, and $\underline{\boldsymbol{G}}_3 \equiv \underline{\boldsymbol{g}}_r$. Measurements of the $\underline{\boldsymbol{G}}_m$ fields will now be described, without loss of substance, by employing: a) The *weak field and coupling approximations;* $\in_m \cong {}^0\in_m$, $\underline{\ell} \cong 1$, and $(\mathbf{w}^0)^2 \cong 1$. And b) The (non italicized) *light-speed normalizations*; $\underline{\mathbf{g}}_r \equiv \underline{\boldsymbol{\zeta}} = \underline{\boldsymbol{g}}_r/c$, $\underline{\mathbf{g}}_0 = \underline{\boldsymbol{g}}_0/c^2$, and $\underline{\mathbf{g}}_s = \underline{\boldsymbol{g}}_s/c^2$. This allows the $\underline{\boldsymbol{G}}_m$ fields to be expressed as the mathematically simpler set, $\underline{\mathbf{G}}_m$(`$\underline{\mathbf{g}}_0$, $\underline{\mathbf{g}}_s$, $\underline{\boldsymbol{\zeta}}$ ), of Maxwellian gravity fields:

$$\underline{\mathbf{g}}_r \equiv \underline{\boldsymbol{\zeta}} = (\underline{\nabla}\times\underline{\mathbf{u}}) \equiv 2\underline{\mathbf{s}}. \tag{3.55b}$$

$$\underline{\mathbf{g}}_0 \equiv \partial\underline{\mathbf{u}}/\partial\underline{x}^0 = \underline{\mathbf{u}}_0\,, \tag{3.56b}$$

$$\underline{\mathbf{g}}_s \equiv \underline{\nabla}(\tfrac{1}{2}\underline{\mathbf{u}}^2) \equiv \underline{\nabla}(\underline{\Phi}_m), \tag{3.57b}$$

### *b. On the qualified use of 3-vector descriptions of $\underline{\mathbf{G}}_m$ sensors.*

In order to simplify mathematical notations, the workings of $\underline{\mathbf{G}}_m$-sensors will be described via the ordinary 3-vector calculus, *as if the* $\underline{\mathbf{G}}_m$ *sensors were inherently 3-dimensional*, which is generally not the case. With only a few exceptions, $\underline{\mathbf{G}}_m$-sensors are designed to measure just one component of each $\underline{\mathbf{G}}_m$ field, and a typical inertial navigation system then employs three mutually orthogonal *one-dimensional* $(\underline{\mathbf{G}}_m)^\alpha$-sensors to measure the mutually orthogonal components of $\underline{\mathbf{G}}_m$ in $\underline{\Sigma}$, which the navigation computer then sums to construct the instantaneous $\underline{\mathbf{G}}_m$ field in $\underline{\Sigma}$. No loss of substance will be entailed if we thus acknowledge that an actual $\underline{\mathbf{G}}_m$ sensor generally senses only one component of the $\underline{\mathbf{G}}_m$ field and the associated $\underline{\mathbf{G}}_m$ forces that are here described with 3–vectors.

3. On the proper measurement of the $\underline{\zeta}$ field via ring-lasers.

*a. The Ring-Laser $\underline{\zeta}$-field sensor, overview.*

A proper measurement of a $\underline{\zeta}$ field in $\underline{\Sigma}$ can be accomplished by means of a *Ring-Laser* (RL) which is stationary in $\underline{\Sigma}$ and which employs three mirrors to confine two counter propagating laser beams to a common propagational path of physical length $\underline{\mathrm{L}}$ enclosing an oriented area, $\underline{\mathbf{A}} = \underline{\alpha}\ \underline{\mathrm{A}}$. The vector $\underline{\alpha}$ then serves to define, via the right-hand curl-rule; the *ccw direction* for both 1) *the propagation* of electromagnetic radiation in the *ccw beam*, and 2) *the circular flow* of $\in_m$ corresponding to a positive value of $\underline{\zeta}\bullet\underline{\alpha}$.

The cw and ccw radiation leaking through the RL's output mirror is combined to form a fringe pattern, and a pair of photo-detector 'eyes' are then employed to electronically determine the + (or –) direction of fringe pattern motion—corresponding to $\underline{\zeta}^{\alpha} = \underline{\zeta}\bullet\underline{\alpha} > 0$ (or $< 0$). Each phase-shift of $2\pi$ radians is then registered as a *directionally-signed electromagnetic pulse.*

Hence, in accordance with the theory of operation described next, the proper signal of a ring laser; is a *signed electromagnetic beat frequency,* $\Delta\underline{\mathrm{f}}^{\alpha}$, that provides a direct and sensitive proper measure of $\underline{\zeta}^{\alpha}$ via the proper response function $\underline{\zeta}^{\alpha} = \Delta\underline{\mathrm{f}}^{\alpha}/\mathrm{K}$.

*b. The proper RL response function, $\Delta\underline{f}^{\alpha} = \mathrm{K}\underline{\zeta}\bullet\underline{\alpha}$.*

The wavelength of the ccw laser beam is fixed by the resonance condition

$$\mathrm{N}\underline{\lambda}_{ccw} = \underline{L}_{ccw} = \oint [\underline{\kappa}(\underline{\mathrm{r}})]_{ccw}\, d\underline{\mathrm{r}}_{ccw}, \tag{3.59}$$

where $[\underline{\kappa}(\underline{\mathrm{r}})]_{ccw}$ is the ccw index of refraction for electromagnetic radiation and $\underline{L}_{ccw}$ is the corresponding ccw *optical length* of the invariant geometric length

$$\underline{\mathrm{L}} = \oint d\underline{\mathrm{r}}_{cw} = \oint d\underline{\mathrm{r}}_{ccw}. \tag{3.60}$$

Here N is an integer N that places $\underline{\lambda} = \underline{L}/\mathrm{N}$ near the peak of the $\underline{\lambda}$–gain-curve for the RL's gain-medium, and the reciprocal of $[\underline{\kappa}(\underline{\mathrm{r}})]_{ccw}$ is the generalized rectilinear speed (GRS) of electromagnetic radiation along the ccw direction of $\underline{\mathrm{L}}$, expressed as a function of both $\underline{\mathbf{u}}$ and the index of refraction of the RL's gaseous gain-medium, $\chi \cong 1$. Thus,

$$1/[\underline{\kappa}(\underline{\mathrm{r}})]_{ccw} = \underline{\beta}^{0}{}_{ccw}\,(\underline{\mathbf{n}}^{0}{}_{ccw}, \underline{\mathbf{u}}, \chi). \tag{3.61}$$

Similar properties and relations are obtained for the cw beam by simply interchanging ccw and cw.

Without loss of substance we will ignore the small and constant contribution effect of $\chi$, and employ Eq. (3.22a)—referenced to $\underline{\Sigma}$ instead of $\grave{}\underline{\Sigma}$—to describe $[\underline{\kappa}(\underline{\mathrm{r}})]_{ccw}$ and $[\underline{\kappa}(\underline{\mathrm{r}})]_{cw}$. Hence, with $\underline{\mathbf{n}}^{0}{}_{cw} = -\,\underline{\mathbf{n}}^{0}{}_{ccw}$, and $\mathrm{N}_{ccw} = \mathrm{N}_{cw} = \mathrm{N}$, the signed electromagnetic frequency ($\Delta\underline{\mathrm{f}}^{\alpha}$) created by a flux of $\underline{\zeta}$ through $\underline{\mathbf{A}}$ is calculated via the following additional definitions and relations:

$$\underline{L}_{ccw} = \oint \underline{\kappa}_{ccw}\, d\underline{\mathrm{r}} = \oint [\underline{\beta}^{0}(\underline{\mathbf{n}}^{0}{}_{ccw}, \underline{\mathbf{u}})]^{-1} d\underline{\mathrm{r}} = \oint \{+ (\underline{\mathbf{n}}^{0}{}_{ccw} \bullet \underline{\mathbf{u}}) + [1 - (\underline{\mathbf{n}}^{0}{}_{ccw} \times \underline{\mathbf{u}})^{2}]^{1/2}\}^{-1} d\underline{\mathrm{r}}. \tag{3.62a}$$

$$\underline{L}_{cw} = \oint \underline{\kappa}_{cw}\, d\underline{\mathrm{r}} = \oint [\underline{\beta}^{0}(\underline{\mathbf{n}}^{0}{}_{cw}, \underline{\mathbf{u}})]^{-1} d\underline{\mathrm{r}} = \oint \{- (\underline{\mathbf{n}}^{0}{}_{ccw} \bullet \underline{\mathbf{u}}) + [1 - (\underline{\mathbf{n}}^{0}{}_{ccw} \times \underline{\mathbf{u}})^{2}]^{1/2}\}^{-1} d\underline{\mathrm{r}}. \tag{3.62b}$$

$$\Delta\underline{\mathrm{f}}^{\alpha} \equiv \underline{\mathrm{f}}_{ccw} - \underline{\mathrm{f}}_{cw} = \frac{c}{\underline{\lambda}_{ccw}} - \frac{c}{\underline{\lambda}_{cw}} = c\mathrm{N}\Big(\frac{1}{\underline{L}_{ccw}} - \frac{1}{\underline{L}_{cw}}\Big) = \frac{c\mathrm{N}}{\underline{L}_{cw}\underline{L}_{ccw}}\big(\underline{L}_{cw} - \underline{L}_{ccw}\big). \tag{3.63}$$

$$\cong \frac{cN}{\underline{L}^2}\left(\underline{L}_{cw} - \underline{L}_{ccw}\right) = \frac{c}{\underline{\lambda L}}\left(\underline{L}_{cw} - \underline{L}_{ccw}\right), \tag{3.64}$$

where $\underline{\lambda}$ is the degenerate lasing wavelength that obtains when $\underline{\mathbf{u}}(\underline{r}) = 0$. Using (3.62) and Stokes' theorem we then obtain $\Delta\underline{f}^{\alpha}$ in the form

$$\Delta\underline{f}^{\alpha} \cong \frac{2c}{\underline{\lambda L}} \oint \underline{\Gamma}^2(\underline{\mathbf{n}}^0{}_{ccw}\bullet\underline{\mathbf{u}})d\underline{r} = \frac{2c}{\underline{\lambda L}} \oiint \underline{\Gamma}^2(\boldsymbol{\zeta}\bullet\underline{\alpha})d(\underline{A}), \tag{3.65a}$$

$$\cong \frac{2c\underline{\Gamma}^2}{\underline{\lambda L}} <(\underline{\varsigma}\bullet\underline{\alpha})>\underline{A} \cong \frac{2c\underline{A}}{\underline{\lambda L}}\boldsymbol{\zeta}^{\alpha} = K\boldsymbol{\zeta}^{\alpha}, \tag{3.65b}$$

where $K = 2c\underline{A}/(\underline{\lambda L})$. This proper $\Delta\underline{f}^{\alpha}$ response of a ring laser to $\boldsymbol{\zeta}^{\alpha}$ therefore permits $\boldsymbol{\zeta}^{\alpha}$ to be quantified via the formula

$$\underline{\boldsymbol{\zeta}} = 2\underline{\boldsymbol{s}} = \Delta\underline{\mathbf{f}}/K. \tag{3.66}$$

4. On the unwitting experimental confirmation of Eq. (3.66)

Recalling the relative GAV equation,

$$`\underline{\boldsymbol{s}}(\in_m/`\underline{\Sigma}) \equiv \boldsymbol{s}(\in_m/\Sigma^d) - \underline{\underline{\omega}}(\underline{\Sigma}/\Sigma^d) = -\underline{\underline{\varpi}}(\underline{\Sigma}/`\Sigma_{\in}), \tag{2.13}$$

and the fact that in deriving (3.66) we set $\Gamma$ to unity, consistent with the restrictions set forth in III.N.2.a, we can safely set,

$$\underline{\boldsymbol{s}}(\in_m/\underline{\Sigma}) \cong `\underline{\boldsymbol{s}}(\in_m/`\underline{\Sigma}), \tag{3.67}$$

and, using (3.66), obtain

$$\Delta\underline{\mathbf{f}}(RL/\underline{\Sigma}) \cong 2K[\boldsymbol{s}(\in_m/\Sigma^d) - \underline{\underline{\omega}}(\underline{\Sigma}/\Sigma^d)]. \tag{3.68}$$

Hence, if the absolute vorticity of $\in_m(x^i)$ due to distant matter can be safely assumed to be negligible, $\Delta\underline{\mathbf{f}}(\underline{RL}/\underline{\Sigma})$ can be assumed to give the rotation rate of the ring laser and the spacecraft, relative to the ${}^0\in_m$–continuum, in accordance with the formula

$$\Delta\underline{\mathbf{f}}(\underline{RL}/\underline{\Sigma}) = 2K\underline{\boldsymbol{s}}(\in_m/\underline{\Sigma}) \cong -2K\underline{\underline{\omega}}(\underline{\Sigma}/\Sigma^d) = -\frac{4\underline{A}c}{\underline{\lambda L}}\underline{\underline{\omega}}(\underline{\Sigma}/\Sigma^d). \tag{3.69}$$

Equation (3.68) therefore covers the formula

$$\Delta\underline{\mathbf{f}}(\underline{RL}/\underline{\Sigma}) = -\frac{4\underline{A}c}{\underline{\lambda L}}\underline{\underline{\omega}}(\underline{\Sigma}/S^0), \tag{3.70}$$

of contemporary ring-laser theory; wherein $S^0$ denotes the traditional concept of *empty inertial space* (${}^0\in_m = 0$), and *rotational motion*, relative in $S^0$, is allowed to break the 3-dimensional isotropy of lightspeed required by special relativity theory. The historically felt need to interpret all observations consistent with ${}^0\in_m = 0$ therefore prohibited theorists from devising a proper (in situ) field theoretic explanation of the proper observation, $\Delta\underline{\mathbf{f}}(\underline{RL}/\underline{\Sigma})$.

Because there are numerous *external ways* to confirm a change in the orientation of a spacecraft, it can be safely stated that the accuracy of Eq. (3.70) has been extensively confirmed over the last 30 years by the *strapped-down* (quasi solid state) technology of inertial navigation which came into being in the early 1970's due to the invention of the ring-laser around 1962.

It then follows that Eq. (3.66), which embodies all of the working ${}^0\mathcal{E}_m$-Implications (i–iv) of Maxwell's gravity theorem, has been experimentally confirmed, albeit unwittingly, for the last three decades by the successful employment of ring lasers as 'rotation sensors' within quasi solid state inertial navigation systems.

*a. The $\Delta\underline{f}^{\alpha}(\underline{\underline{\omega}})$ scale factor for a typical navigation grade RL.*

Given the response function of (3.69), a typical navigation grade ring-laser (characterized by $\underline{\lambda}$ = 0.63 microns, $\underline{L}$ = 30 cm, $\underline{A}$ = 43 cm$^2$) generates 3,176 electromagnetic pulses (per second) for each degree (per second) of $\underline{\Sigma}$'s angular propagation in the ${}^{0}\in_{m}$-continuum.[21] This $\underline{\underline{\omega}}$-proportional *frequency* sensitivity is a factor of c/$\underline{L}$ = $10^9$ times the $\underline{\underline{\omega}}$-proportional *fringe-shift* sensitivity of a physically equivalent Sagnac interferometer.

5. On the $\in_m$-field unification of 'elevator' gravity and Newtonian gravity.

*a. A restrained-mass (RM) instrument.*

A restrained-mass (RM) instrument is an instrument which develops whatever restraining force, $\underline{\mathbf{F}}_R = -E^0\underline{\mathbf{g}}$, is required to prevent an otherwise free $c^\pi$ particle, of rest energy $E^0 = m^0c^2$, from moving in response to a proper Maxwellian gravitational field, $\underline{\mathbf{g}}_m$, which, like $\underline{\mathbf{g}}_0$ and $\underline{\mathbf{g}}_s$, has the units of acceleration per unit $c^2$.

*b. On the proper measurement of* $\underline{\mathbf{g}}_0$.

Recalling the relative GRA equation

$$`\underline{\mathbf{u}}_0(\in_m(`\underline{\mathbf{r}})/`\underline{\Sigma}) = [\mathbf{u}_0(\in_m(`\underline{\mathbf{r}})/\Sigma^d) - \underline{\underline{\beta}}_0(\underline{\Sigma}/\Sigma^d)] = -\underline{\underline{\mathbf{w}}}_0(\underline{\Sigma}/`\underline{\Sigma}_\epsilon), \tag{2.17}$$

and the restrictions set forth in III.N.2, we can safely set,

$$\underline{\mathbf{g}}_0(\in_m(\underline{\mathbf{r}})/\underline{\Sigma}) = \underline{\mathbf{u}}_0 \cong `\underline{\mathbf{u}}_0(\in_m(`\underline{\mathbf{r}})/`\underline{\Sigma}),$$

and thereby obtain

$$\underline{\mathbf{g}}_0(\in_m(\underline{\mathbf{r}})/\underline{\Sigma}) = [\mathbf{u}_0(\in_m(\underline{\mathbf{r}})/\Sigma^d) - \underline{\underline{\beta}}_0(\underline{\Sigma}/\Sigma^d)] = -\underline{\underline{\mathbf{w}}}_0(\underline{\Sigma}/`\underline{\Sigma}_\epsilon). \tag{3.71}$$

Hence, when it can be further safely assumed that $\mathbf{u}_0(\in_m/\Sigma^d) = \mathbf{0}$—which is true for the kinds of spacecraft mission environments typically encountered to date (no black-hole or cosmic-string missions, as yet)—then $\underline{\mathbf{F}}_R$ can be safely assumed to be a proportionate measure of the rectilinear acceleration of $\underline{\Sigma}$ relative to the ${}^{0}\in_{m}$-continuum; in accordance with the equation

$$\underline{\mathbf{F}}_R(\underline{x}^0)/E^0 = -\underline{\mathbf{g}}_0(\in_m(\underline{\mathbf{r}})/\underline{\Sigma}) = \underline{\underline{\beta}}_0(\underline{\Sigma}/\Sigma^d) = \partial^2\underline{\underline{\mathbf{r}}}/(\partial x^0)^2. \tag{3.72}$$

Equation (3.72) therefore covers the formula

$$\underline{\mathbf{F}}_R(\underline{x}^0)/E^0 = \underline{\underline{\beta}}_0(\underline{\Sigma}/S^0) = \partial^2\underline{\underline{\mathbf{r}}}/(\partial x^0)^2. \tag{3.73}$$

of contemporary inertial sensor theory; wherein $S^0$ denotes the traditional concept of *empty inertial space* (${}^{0}\in_{m} = 0$), and $\underline{\mathbf{F}}_R$ is the Newtonian force required in $\underline{\Sigma}$—on account of the unexplained inertia of $E^0$—to give the $c^\pi$ particle in $\underline{\Sigma}$ the same acceleration relative to $S^0$ as the RM instrument in $\underline{\Sigma}$. The historically felt need to interpret all observations consistent with ${}^{0}\in_{m} = 0$ therefore prohibited inertial sensor theorists from devising a natural proper (in situ) field theoretic explanation of the proper observation, $\underline{\mathbf{F}}_R(\underline{RM}/\underline{\Sigma})$.

A navigation computer upon receiving $\underline{\mathbf{F}}_R/E^0$, as a digital electronic signal, integrates it twice with respect to time to obtain the corresponding changes in $\underline{\Sigma}$'s $\underline{\underline{\beta}}$ and $\underline{\underline{\mathbf{r}}}$ relative to their initially prescribed values. And since there are numerous *external ways* to confirm these changes, it can be safely stated that the accuracy of Eq. (3.73) has been extensively confirmed over the last 40 years by the technology of inertial navigation which came into being in the early 1950's—about ten years before the invention of the ring laser, employing mechanical gyroscopes to determine the changes in $\underline{\Sigma}$'s attitude.

*It follows that Eq. (3.72) has been experimentally confirmed, albeit unwittingly, for the last four decades.*

*c. The delimiting attributes of ideal Newtonian gravity.*

Ideal Newtonian gravity is here defined with respect to $`\underline{\Sigma}$ as being sourced by a spherically symmetric distribution of electrically neutral mass-energy, $E^0_n$, which is centered on $`\underline{O}$, occupies a 3-volume of radius $`\underline{R}$, and is subject to the null-propagation constraints $\mathbf{w}(E^0_n) = \omega(E^0_n) = \mathbf{0}$.

The field point $`\underline{\mathbf{r}} > `\underline{\mathbf{R}}$ is seen from the perspective of $\Sigma^d$ as the field point $\mathbf{r} = `\underline{\underline{\mathbf{r}}} + `\underline{\mathbf{r}}$, where $`\underline{\underline{\mathbf{r}}}$ is the fixed vector distance from O to $`\underline{O}$. Because of the *null-propagation* status of $`\underline{\Sigma}$, the *relative generalized* $\in_m$-fields, $`\underline{\mathbf{u}}$, $`\underline{\mathbf{s}}$, and $`\underline{\mathbf{u}}_0$, are identical to the corresponding *absolute* $\in_m$–fields $\mathbf{u}$, $\mathbf{s} = ½\underline{\boldsymbol{\zeta}} = ½(\nabla\times\mathbf{u})$, and $\mathbf{u}_0 = \partial\mathbf{u}/\partial x^0$. These conditions cause the $\in_m$–particle coupling at $`\underline{\mathbf{r}}$ of $`\underline{\Sigma}$ and $\mathbf{r}$ of $\Sigma^d$ to be restricted to the purely Γ-coupling described in III.A.2. Hence we can here dispense with the pre-prime notation for the $\in_m$-fields *per se,* and describe, unambiguously, the absolute $\in_m$-fields at $`\underline{x}^i$ of $`\underline{\Sigma}$—thereby retaining the pre-prime notation only for the un-deformable reference system $`\underline{\Sigma}$.

The absolute force, $`\underline{\mathbf{F}}_n(`\underline{\mathbf{r}})$, of ideal Newtonian gravity on a stationary test particle, $E^0$, at $`\underline{\mathbf{r}} > `\underline{\mathbf{R}}$, can then be unambiguously described by:

$$`\underline{\mathbf{F}}_n(`\underline{\mathbf{r}}) = E^0\underline{\mathbf{g}}_n(`\underline{\mathbf{r}}) = -GE^0E^0_n\,`\underline{\mathbf{r}}/`\underline{r}^3, \tag{3.74a}$$

$$\equiv -(E^0)`\underline{\nabla}(\Phi_n/c^2), \tag{3.74b}$$

where,

$$\Phi_n/c^2 \equiv -GE^0_n/`\underline{r} = -\mathcal{G}E^0_n/`\underline{r}c^4, \tag{3.75}$$

$G$ is the gravitational constant ($6.67\text{x}10^{-11}$ m$^3$/(Kg-sec$^2$)), and the quantity $(-E^0\Phi_n/c^2)$ is both the Newtonian gravitational potential energy of $E^0$ (relative to $`\underline{\Sigma}$), and, the *mass-energy defect* or *binding energy* of the stationary ($E^0E^0_n$) particle system.[a]

The *acceleration field* of ideal Newtonian gravity, $\mathbf{g}_n(`\mathbf{r})$, is then quantified via Eqs. (3.74) as

$$\underline{\mathbf{g}}_n \equiv -`\underline{\nabla}[\Phi_n(`\underline{\mathbf{r}})/c^2] = `\underline{\mathbf{F}}_n(`\underline{\mathbf{r}})/E^0, \tag{3.76a}$$

and for the weak field-coupling approximation this is the acceleration field

$$\underline{\mathbf{g}}_n \cong \mathbf{g}_n = -\mathbf{F}_R/E^0 \tag{3.76b}$$

that would be quantified by a stationary RM instrument ($\underline{\Sigma}$) at `r, functioning as a *gravimeter* instead of an 'elevator' gravity sensor or $\underline{\Sigma}$ *accelerometer*.

*d. The $\in_m$-field explanation of ideal Newtonian gravity.*

Because an RM instrument can not distinguish between the $\underline{\mathbf{g}}_n$ field of ideal Newtonian gravity and the $\underline{\mathbf{g}}_0$ $\in_m$-field 'elevator' gravity, we must conclude that $\underline{\mathbf{g}}_n$ and $\underline{\mathbf{g}}_0$ are just different aspects of the same physical thing—a principle, commonly referred to as the *acceleration equivalence principle,* that has long been recognized and pragmatically exploited, but never theoretically explained. From the perspective of Maxwellian gravity then, we are justified in predicting that an RM instrument in $\underline{\Sigma}$ responds to a more general relative $\in_m$-acceleration field

$$\underline{\mathbf{g}}_m = \underline{\mathbf{g}}_0 + \underline{\mathbf{g}}_s, \tag{3.77}$$

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[a] The mass defect explanation of $-E^0\Phi_n/c^2$ suggests that; in describing the orbits of attractively bound particles, allowance must be made for the decreases in the rest energy (and thus inertia) of each of the particles.

where $\mathbf{g}_s$ is the $\in_m$-field explanation of $\mathbf{g}_n$. And since $\mathbf{g}_0$ represents the *explicit* time-rate-of-change of $\mathbf{u}(x^i)$, we are encouraged to seek a common explanation for both $\mathbf{g}_0$ and $\mathbf{g}_n$ in the *more general acceleration field* of $\in_m$ described by the *total* time-rate-of-change of $\mathbf{u}(x^i)$. We therefore apply a well known fluid dynamic equation to obtain the Maxwellian expression

$$\mathbf{g}_m \equiv d\underline{\mathbf{u}}/d\underline{x}^0 = \partial\underline{\mathbf{u}}/\partial\underline{x}^0 + (\underline{\mathbf{u}}\bullet\nabla)\mathbf{u}, \tag{3.78a}$$

$$= g_0 + \nabla(\tfrac{1}{2}\mathbf{u}^2) - \mathbf{u}\times(\nabla\times\mathbf{u}), \tag{3.78b}$$

$$\equiv \mathbf{g}_0 + \mathbf{g}_s - \mathbf{u}\times\boldsymbol{\zeta}, \tag{3.78c}$$

for the total (temporal and spatial) acceleration of $\in_m$. Which leads us to conclude that; to the degree of approximation consistent with both the conservative ($\boldsymbol{\zeta} = \mathbf{0}$) and weak-field characteristics of ideal Newtonian gravity, the $\mathbf{g}_m$-field of Maxwellian gravity—embracing 'elevator gravity and ideal Newtonian gravity—is both described and explained by equation

$$\mathbf{g}_m = \mathbf{g}_0 + \mathbf{g}_s = \partial\mathbf{u}/\partial x^0 + \nabla(\tfrac{1}{2}\mathbf{u}^2) \equiv \partial\mathbf{u}/\partial x^0 + \nabla(\Phi_m). \tag{3.79}$$

Thus, in addition to providing an intuitive physical explanation for 'elevator' gravity, Maxwellian gravity offers a fundamental explanation for the scalar potential of ideal Newtonian gravity, via the equalities

$$\Phi_n/c^2(r) = -\Phi_m = -\tfrac{1}{2}\mathbf{u}^2(r) = -GM_n/\underline{r}c^2, \tag{3.80a}$$

and,

$$[1 + 2\Phi_n/c^2(r)] = [1 - 2\Phi_m] = [1 - \mathbf{u}^2(r)] = 1/\Gamma^2. \tag{3.80b}$$

The potentials ($\Phi_m$ and $\mathbf{u}$) of Maxwellian gravity exhibit an interesting *pseudo 4-potential* analogy to the 4-potential of Maxwell's theory of electrodynamics. This analogy is summarized next.

*e. The pseudo 4-potential of Maxwellian gravity.*

In accordance with (3.79) and (3.80a), the expression (3.74b) for the force of Newtonian gravity, can be usefully replaced by the more general expression

$$\mathbf{F}_m(x^i) = E^0\mathbf{g}_m(x^i) = E^0(\nabla\Phi_m + \partial\mathbf{u}/\partial x^0), \tag{3.81}$$

for the force of Maxwellian gravity which is exerted on a $p^\pi$ particle that is instantaneously at rest at $x^i$ of $\Sigma^d$, due to both the presence and the propagation dynamics of the energy of a distant $c^\pi$ particle. This Maxwellian *gravity force* happens to be functionally identical to the *gravity-free* ($\mathbf{u} = \mathbf{0}$) Maxwellian *electromagnetic force* which is exerted on an a negative charge ($q_-$) that is instantaneously at rest at $x^i$ of $\Sigma^d$, due to both the presence and the propagation dynamics of the electric charge (Q) carried by a distant $p^\pi$ particle. Namely, the force

$$\mathbf{f}_{q-}(x^i) = -q_-\mathbf{E}(x^i) = q_-[\nabla\varphi_e + \partial\mathbf{A}^*/\partial x^0], \tag{3.82}$$

where, as discussed in Section III.E.1.a, $\varphi_e(x^i)$ and $\mathbf{A}^*(x^i) = \mathbf{c}\mathbf{A}(x^i)$ are the components of the electrodynamic 4-potential $\varphi_e^i(x^i) = (\varphi_e, \mathbf{A}^*) = \varphi_e(x^i)(1, \beta) \equiv \varphi_e\beta^i$.

Hence, to the degree of approximation with which we are presently working, we assert that

$$\Phi_m^i(x^i) = (\Phi_m, \mathbf{u}), \tag{3.83}$$

is a pedagogically useful, *pseudo 4-potential* of Maxwellian gravity, that is to $E^0$ what $\varphi_e^i(x^i) = (\varphi_e, \mathbf{A}^*)$ is to negative charge—knowing full well that this assertion will be both confirmed and clarified by the consequences of EM gravity in Section V—although no effort has been expended as yet to show that, either like (or unlike) **A***, a co-propagating (1/r) **u**-field is (or is not) induced in the $^{0}\in_{m}$–continuum by the GRV of $E^0$. If it is, its mathematical expression could be modeled in strict analogy with the **A*** field of propagating electric charge, and possibly confirmed via ring-laser `ζ-field sensors.

*f. The fundamentality of field-potentials—expanded.*

The fact that the pseudo 4-potentials, $\Phi^{i}_{m}(x^{i}) = (\Phi_m, \mathbf{u})$, of Maxwellian gravity are clearly more fundamental than the readily observable gravity fields resulting from their temporal and spatial variations, harmonizes with the late 20th century revelation of quantum mechanics (pointed out in Section III.E.1.a); that the 4-potentials $\varphi^{i}_{e}(x^{i}) = (\varphi_e, \mathbf{A}^*)$ are in fact more fundamental than the readily observable electromagnetic fields resulting from their temporal and spatial variations—thereby extending the range of validity of the quantum mechanical revelation. Only now it is clear that all field potentials are *not equally fundamental*, and that the 3-vector potential

$$\mathbf{p}_m(x^{i}) = \in_m(x^{i})\, \mathbf{u}(x^{i}), \tag{2.2}$$

of Maxwellian gravity—being a generator of three, interrelated, gauge-field theories (GFT1-3), which lead unerringly to the hyper-energy field equations governing the creation and the evolution of compactified hyper-dimensional universes and their particles—is, in fact, the *most fundamental field-potential.* This claim is in perfect accord with Robert Mills' assertion that:

> "There seems to be little doubt now that the ultimate theory, if it is ever accurately identified, will turn out to be gauge theory. My own feeling it that there will have to be at least one more major conceptual revolution before that final goal is achieved."[22]

This revelation; that gravity is more fundamental than other matter fields—despite its "relative weakness"—helps to explain its long recognized overriding importance as *the energy underlying material evolution*. It thus follows that ideal Newtonian gravity and 'elevator' gravity are simply the first two classical but readily quantizable matter fields to fall under the purview the new $\in_m$-physics, and for these reasons it will come as a very great and very welcome surprise to learn that, contrary to present fears,[23] gravity will be, in fact, *the simplest* rather than *the most complicated* field to absorb into a unified *classical-quantum* theory of particle structure, consistent with all known particle interactions and transmutations.

## IV. Einstein-Maxwell Gravity.

### **A.** Einstein's foresight

It is well known that Einstein never wavered from his conviction that the unification of cosmology and particle physics would come about via a continuum theory that would be able to account for cosmology and the structures of *all* particles. Einstein defended this proposition with enthusiasm in his correspondences—referring to his own efforts to bring this about, interestingly enough, as *his* "Maxwellian Program". (Holton, 1996)

In (Holton, 1996) we "*hear"* Einstein arguing that *we can not reasonably settle for a wave-particle duality in nature* that gives more or less *equal status* to both *the field* and to its antithesis*, discrete matter-energy*. And for Einstein, there was no question as to which of these two extremes had the greatest logical probability of being both necessary and sufficient to explain everything. As he said in a letter to his old friend, Michele Besso,

> "I consider it quite possible that physics might not, finally, be founded on the concept of field—that is to say, on continuous elements. But then out of my whole castle in the air—including the theory of gravitation, but also most of current physics—there would remain *nothing.*" (Holton, 1996)[24]

Einstein also believed that any such classical field theory and cosmology would necessarily incorporate his general theory of relativity in some manner.

### **B.** The mass-energy equivalence principle and tensor mathematics furnished by Einstein.

#### 1. Einstein's $E = mc^2$ unification of particle energy and mass.

Einstein's unification in 1905 of the two previously disparate physical concepts of particle energy and particle mass greatly enlarged the then fairly new and restricted 19th century conception of energy; endowing energy with a *universality* and *objective substance* that was previously unknown.

The entire domain of matter particles and fields began to be viewed thereafter in an entirely new fundamental way: As different manifestations of a universal type of *energy,* called *mass-energy*, which exhibits the *mechanical property* of inertia that, previously, only *bodies of matter* possessed. Thus making it possible to conclude that, in retrospect:

> *Energy* is a more fundamental *classical substance* for the discipline of *fluid mechanics* than the passive and inert property of *mass* on which fluid mechanics—the very model and inspiration for *field theory,* to begin with—was originally founded.

The fact that all matter particles and fields consist of *mass-energy*, is in qualitative agreement with implication (i) of Maxwell's SE Gravity theorem; that the $\delta\mathcal{E}_m$ energy of any matter particle or field represents a miniscule perturbation of ${}^0\mathcal{E}_m$, denoted symbolically by $\delta\mathcal{E}_m$.[25] One can thus assume that the $(1 + p)$-dimensional *hyper-energy* explanations for ${}^0\mathcal{E}_m$, its $(\delta\mathcal{E}_m)$s, and thus, $\delta\mathcal{E}_m$ propagation, will explain both the $(1 + 3)$ *mass* (m) and associated $mc^2$ *energy* of any $\delta\mathcal{E}_m$ in full agreement with the implication (iv); that $c^2$ is an indirect measure of ${}^0\in_m$. These assumptions are qualitatively vindicated with the compactification model described in Sections VII – IX.

**2.** The (1 + 3) tensor calculus of covariant and generally covariant physical laws.

At the turn of the $20^{th}$ century, the labors of a number of mathematicians throughout the last half of the $19^{th}$ century had produced the elements of a *static* p−dimensional tensor calculus.[26] And Minkowski deserves credit for pointing out, in 1908, that special relativity theory could be elegantly presented and utilized in the context of a 4-dimensional *geodynamic* (3 + 1) subset of the tensor calculus.[27]

Einstein's special theory of relativity can thus be rightfully credited with causing a unique and powerful (1 + 3) branch of the (0 + *n*)−dimensional tensor calculus to become established as an inviolably economical and practical tool for developing descriptions of *mass-energy physics*—in all of its many diverse domains of expression; from high energy particle physics to the nonlinear fluid thermodynamics of numerous practical *mass-energy* continua—descriptions which are guaranteed to be intrinsically Lorentz invariant, and thus, as correct as they can possibly be—at the (1 + 3) level of approximation of physical reality. And of course Einstein himself found the (1 + 3) tensor calculus useful for addressing the problem of gravity in a manner consistent with his perceived ($^0\in_m =\ 0$) view of 3-space.

Hence, the only thing that could make it better would be if general relativity theory—which is known to be actually *too general*—could be mathematically restricted in a way that would make the resulting descriptions completely compatible with Maxwell's gravity theorem. If this could be done it would create a clear (1 + 3)-dimensional Einstein-Maxwell interface to a (1 + *p*)-dimensional hyper energy physics of the future that would bring closure to the goal that Einstein and Maxwell both shared; for a basically *classical* unified field theory. A goal which Einstein and Maxwell laid complementary mathematical foundations for via their respective mathematical theories of electromagnetism and gravity. The remainder of this section will show that this grand EM unification is not only possible but quite inevitable.

**C.** The *implicit* 4-vector potential of gravity defined by general relativity theory.[28]

The general relativistic analysis of gravity is founded on the (1 + 3) tensor formula

$$ds^2 = g_{ik}dx^i dx^k = dx^i dx_k, \tag{4.1}$$

where the $g_{ik}(x^i)$ are presumed to contain 10 unknown *gravitational potentials*—sourced by the 3−volume density of mass-energy and mass-energy-momentum ($T_{ik}$), in accordance Einstein's equations,

$$R_{ik} - ½Rgik = CT_{ik}. \tag{4.2}$$

This wholly *metric* description of gravity happens to incorporate an *inherently dynamical* 4−vector potential which surfaces when Eq. (4.1) is put into the following equivalent *dynamical* form:

$$ds^2 = g_{ik}dx^i dx^k = \beta^i\beta_i\,(dx^0)^2, \tag{4.3}$$

where,

$$\beta^i = dx^i/dx^0 = (1, \beta), \text{ and } \beta_i = dx_i/dx^0 = (\beta_0,\ \beta_\alpha)\ = (\beta_0, \vec{\beta}\ ).^{a} \tag{4.4}$$

From (4.3) and (4.4), it can be seen that *all* of the field-theoretic information about the field of gravity—for any conceivable physical situation (irrespective of how the field is sourced)—is stored in nothing more than, and nothing less than, the four elements of

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[a] As indicated, we will be using an *over arrow* to denote the *3-vector* part of a *covariant* 4−vector.

$$\beta_i = (\beta_0, \beta_\alpha) = g_{ik}\beta^k.^{29} \tag{4.5}$$

The particular way that the gravity potentials are stored in $\beta_i$ can be seen most clearly by defining $g^\alpha = g_\alpha = g_{0\alpha}$, and $G^\alpha = G_\alpha = g_{\alpha\gamma}\beta^\gamma$, so that the components of $\beta_i$ can be expressed as

$$\beta_0 = (g_{00} + \mathbf{g}\bullet\beta), \tag{4.6}$$

$$\vec{\beta} = (\mathbf{g} + \mathbf{G}). \tag{4.7}$$

Hence, all of the (1 + 3) information about the gravitational potentials—for *any conceivable gravitational field*—is contained in this particularly simple looking mathematical structure. It thus seems clear that the completion of Einstein's Maxwellian program and a solution to quantum gravity problem can now be precipitated by simply

- Deriving an explicit expression for $\beta_i$ which is uniquely consistent with Maxwell's gravity theorem.
- Allowing the resulting *dynamical representation* of $ds^2$ (the right side of (4.3)) to be the *fundamental scalar* that the *geometrical representation* (the left side of (4.3)) is slaved to.

This is precisely what is done next.

### D. The dynamical scalar illuminated by flat spacetime.

In the $\gamma$–coupling domain of the $\in_m$-continuum, Eq. (4.3) becomes

$$ds^2 = G_{ik}dx^i dx^k = (\beta_i\beta^i)(dx^0)^2 = (1 - \beta^2)(dx^0)^2 \equiv (dx^0/\gamma)^2 = [(dx^0)^2 - d\mathbf{r}^2], \tag{4.8}$$

where $\beta_i = (1, -\beta)$, $g_{ik} \rightarrow G_{ik} \equiv \text{diag.}\,(1,-1,-1,-1)$, and $\gamma \equiv 1/[1 - \beta^2]^{½}$.

From the practical perspective of $\Sigma$, ds is a valid 4-scalar because it attains an absolute countable value of zero for the special case that d**r** refers to a point of electromagnetic radiation—as required by Maxwell's theory of electrodynamics. And the successes of Einstein's special relativity theory have assured us that the 4-scalar status of ds is preserved, as one might logically expect, when d**r** refers to a $p^\pi$ particle as well.

#### 1. The expanded (1 + 3) dynamical symmetry underlying Einstein-Maxwell gravity.

The question then is, how can the *dynamical symmetry* of $\beta^i\beta_i$ be expanded so that it converges to $(1 - \beta^2)$ for $\mathbf{u} = \mathbf{0}$, and is otherwise in agreement with **w-**gauge theory? The answer seems to be, simply substitute $\ell$ for $\gamma$ in (4.8) so that $\beta^i\beta_i$ has the explicit form

$$\beta^i\beta_i(p) = (1 - \mathbf{w}^2) \equiv 1/\ell^2, \tag{4.9a}$$

$$= 0 \text{ if } \mathbf{w}(p) \rightarrow \mathbf{w}(p^0) \equiv \mathbf{w}^0, \tag{4.9b}$$

$$\geq 0 \text{ if } \mathbf{w}(p) \rightarrow \mathbf{w}(p^\pi) \equiv \mathbf{w}. \tag{4.9c}$$

The *propagation velocity* of $\delta\mathcal{E}_m$ field-particles in the $\in_m$-continuum then takes over the symmetry role that was previously played by the *ordinary* rectilinear velocity $\beta$ of matter particles in *empty 3–space*. Now that $\beta$, in accordance with Eq. (2.3), is physically explained by *either one* or *both* of two fundamental transport mechanisms (as $\beta = \mathbf{w} + \mathbf{u}$), $\beta^2$ is no longer fixed at unity for a $p^0$ field-particle. Hence something else is needed to create an absolute countable null value for ds when electromagnetic radiation is being described. And **w**—with its null 3-scalar characterization of ${}^0\in_m$ (per Section II.D.3)—seems to be perfectly suited for doing just that.

From the viewpoint of $\Sigma^d$ then, $\mathbf{w}(p) = \beta(p) - \mathbf{u}(x^i)$ describes the 3-vector difference between the coordinate velocity of $p(x^i)$ and the flow velocity of $\in_m$ at $(x^i)$. Thus, wherever it is absolutely necessary or heuristically useful to do so, we will expand **w** into $\beta - \mathbf{u}$, $\underline{\mathbf{w}}$ into $\underline{\beta} - \underline{\mathbf{u}}$, and $\underline{\underline{\mathbf{w}}}$ into $\underline{\underline{\beta}} - \underline{\underline{\mathbf{u}}}$. Otherwise we will exploit the compactness of w, $\underline{\mathbf{w}}$, and $\underline{\underline{\mathbf{w}}}$, and their various inner products in the equations which evolve as we proceed to explore the consequences of employing Eq. (4.9a) as a driving dynamical symmetry for EM gravity.

2. The *explicit* 4-vector potential of Einstein-Maxwell gravity.

To determine the $g_{ik}(\Phi^i)$ consistent with Eq. (4.9a) we set

$$\beta^i\beta_i(p) = [1 - (\beta - \mathbf{u})^2] = [(1 - \mathbf{u}^2) + 2\mathbf{u}\bullet\beta - \beta^2\,], \tag{4.10}$$

and compare this with the right sides of Eqs. (4.6-7) to obtain the unique Einstein-Maxwell components of $\beta_i$ as,

$$\beta_0 = [(1 - \mathbf{u}^2) + \mathbf{u}\bullet\beta)] = (g_{00} + \mathbf{g}\bullet\beta). \tag{4.11a}$$

$$\vec{\beta} = (\mathbf{u} - \beta) = -\,\mathbf{w} \quad = (\mathbf{g} + \mathbf{G}). \tag{4.11b}$$

Hence,

$$g_{oo} = (1 - \mathbf{u}^2) \equiv (1 - 2\Phi_m) = (1 + 2\Phi_n/c^2), \tag{4.12a}$$

$$\mathbf{g} = \mathbf{u}, \tag{4.12b}$$

$$\mathbf{G} = -\,\beta, \tag{4.12c}$$

$$g_{\alpha\gamma} = -\,\delta_{\alpha\gamma}. \tag{4.12d}$$

In matrix form:

$$g_{ik} = \begin{array}{cc} i\ j\rightarrow & \begin{array}{cccc} 0 & \quad 1 & \quad 2 & \quad 3 \end{array} \\ \begin{array}{c} 0\\ 1\\ 2\\ 3 \end{array} & \begin{bmatrix} 1-\mathbf{u}^2 & u^1 & u^2 & u^3 \\ u^1 & -1 & 0 & 0 \\ u^2 & 0 & -1 & 0 \\ u^3 & 0 & 0 & -1 \end{bmatrix} \end{array}, \tag{4.13a}$$

$$g^{ik} = \begin{array}{cc} i\ j\rightarrow & \begin{array}{cccc} 0 & \qquad 1 & \qquad 2 & \qquad 3 \end{array} \\ \begin{array}{c} 0\\ 1\\ 2\\ 3 \end{array} & \begin{bmatrix} 1 & u^1 & u^2 & u^3 \\ u^1 & (u^1)^2-1 & u^1u^2 & u^1u^3 \\ u^2 & u^2u^1 & (u^2)^2-1 & u^2u^3 \\ u^3 & u^3u^1 & u^3u^2 & (u^3)^2-1 \end{bmatrix} \end{array}, \tag{4.13b}$$

where, $g = |g_{ik}| = |g^{ik}| = -1$.

Hence, given $g_{ik}(u^\alpha)$ and the EM 4-GRV

$$\nu^i = dx^i/ds = (\nu^0, \nu) = \ell\beta^i, \tag{4.14}$$

the many consequences of EM gravity can then be derived from the perspective of $\Sigma^d$, via the equation

$$ds^2 = (1 - \mathbf{w}^2)(dx^0)^2 = g_{ik}(u^\alpha)dx^i dx^k = dx^i dx_k = \beta^i\beta_i\,(dx^0)^2 = \nu^i\nu_i\,ds^2 \equiv (dx^0/\ell)^2. \tag{4.15}$$

Operationally it is clear that this equation merely dictates the form of an invariant *4-rate of change operator*, $d/ds = \ell d/dx^0$, which, in turn, dictates the contravariant and covariant forms of the equations describing the first and second order, nonlinear thermodynamics of a free $p^{\pi}$ particle in an EM gravity field parameterized by $g_{ik}(u^{\alpha})$, irrespective of the actual distribution of $\mathbf{u}(x^i)$—*analogous to the source free equations of Maxwell's electrodynamics*.

In the view of Maxwell's equations and the lightspeed scalar, $S_3$, it is also clear that the invariance of $[(dx^0)^2 - (d\mathbf{r})^2]$ for free electromagnetic energy and $p^{\pi}$ particles expressed by

$$ds^0 = d\underline{s}^0 = 0, \tag{4.16a}$$

$$ds = d\underline{s} \geqslant 0, \tag{4.16b}$$

serve as a quantification of NPA–C2.

E. The general (1 + 3) EM **w-**deformation of scalar time measures.[a]

Let $\underline{\underline{v}}^i = \underline{\underline{\ell}}\underline{\underline{\beta}}^i$ be the 4–GRV of both $\underline{O}$ and $\underline{\mathbf{r}}$. The reference point $`\underline{\mathbf{r}}$ and a $\delta\mathcal{E}_m$ particle at $x^i$ of $\Sigma^d$ are then embedded in the same $\in_m$ flow field, so that formally we can set $\underline{\mathbf{u}} = \mathbf{u}$, $\underline{\underline{\mathbf{w}}} = \underline{\underline{\beta}} - \mathbf{u}$, and $\mathbf{w} = \beta - \mathbf{u}$. The empirical time change $d\underline{x}^0$ can then be unambiguously expressed in terms of $\Sigma$ parameters via the *velocity-coupling pseudo scalar* ($\underline{\underline{v}}_i\beta^i = \underline{\underline{v}}^i\beta_i$) via the equation

$$d\underline{x}^0 = (\underline{\underline{v}}_i\beta^i)dx^0 = \underline{\underline{v}}_i dx^i = \underline{\underline{\ell}}(1 - \underline{\underline{\mathbf{w}}}\bullet\mathbf{w})dx^0, \tag{4.17}$$

$$= \underline{\underline{\gamma}}(1 - \underline{\underline{\beta}}\bullet\beta)dx^0 \quad \text{for } \mathbf{u} = \mathbf{0}, \tag{3.33}$$

$$= \underline{\underline{\gamma}}(dx^0 - \underline{\underline{\beta}}\bullet d\mathbf{r}), \tag{3.33}$$

which reduces in the $\gamma$–coupling domain, as just shown, to the temporal component of a general Lorentz transformation that was previously derived from $\underline{\underline{w}}d^{1-3}$ of GFT1 for $\mathbf{u} = \mathbf{0}$.

F. The general (1 + 3) EM **w-**deformation of scalar distance measures.[a]

Given (4.15), (4.17), and the (1 + 3) scalar equality $ds = d\underline{s}$, we obtain $(d\underline{r})^2$ in terms of $\Sigma$ parameters from the equation

$$(d\underline{\mathbf{r}})^2 = (d\underline{x}^0)^2 - (ds)^2 = (\underline{\underline{\beta}}\bullet d\underline{x}^0)^2, \tag{4.18a}$$

$$= (\underline{\underline{v}}_i\,\underline{\underline{v}}_k - g_{ik})dx^i dx^k \equiv m_{ik}\,dx^i dx^k, \tag{4.18b}$$

which evaluates to the expression

$$(d\underline{\mathbf{r}})^2 = [\underline{\underline{\ell}}\,(d\mathbf{r}_{\|} - d\underline{\mathbf{r}})]^2 + (d\mathbf{r}_{\perp})^2, \tag{3.17f}$$

previously derived via GFT1 as one expression of $\underline{\underline{w}}d^1$.

G. The general (1 + 3) EM **w-**deformation of scalar GRV's.

Given (4.17) and (4.18) it can be shown that

$$(\underline{\beta})^2 = \frac{[(\mathbf{w} - \underline{\underline{\mathbf{w}}})^2 - (\mathbf{w}\times\underline{\underline{\mathbf{w}}})^2]}{(1 - \mathbf{w}\bullet\underline{\underline{\mathbf{w}}})^2}, \tag{4.19a}$$

$$= \frac{[(\beta - \underline{\underline{\beta}})^2 - (\beta\times\underline{\underline{\beta}})^2]}{(1 - \beta\bullet\underline{\underline{\beta}})^2} \quad \text{for } \mathbf{u} = \mathbf{0}. \tag{4.19b}$$

Giving

[a] See V.D for the general (1 + 3) EM $\underline{\underline{\mathbf{w}}}$-deformations of arbitrary (1 + 3) tensor quantities at $\underline{O}$ of $\underline{\Sigma}$.

$$\underline{\beta} \;=\; \frac{[\,(\beta_{\|} - \underline{\underline{\beta}}) - \beta_{\perp}/\underline{\underline{\gamma}}]}{(1 - \beta\bullet\underline{\underline{\beta}}\,)} \text{ for } \mathbf{u} = \mathbf{0}, \tag{3.34a}$$

as previously derived via GFT1.

## **V.** Applications and Implications of EM Gravity.

—Contents—



### **A.** The first order EM equations of $p^{\pi}$ particle dynamics.

#### **1.** The structures and scalar associated with fundamental EM 4-velocities.

From Eq. (4.15) I obtain the four-scalar differential operator

$$(d/ds) = \ell d/dx^{0}, \text{ with } \ell \equiv 1/[1 - \mathbf{w}^{2}]^{1/2}, \tag{5.1}$$

and via this the following first-order 4-velocity vectors and associated scalars:

$$dx^{i}/ds \equiv \nu^{i} = (v^{0}, \nu) = \ell\beta^{i} = \ell(1, \beta) \tag{4.14}$$

$$\nu_{i} = g_{ik}\nu^{i} = dx_{i}/ds = (v_{0}, \nu) = \ell\beta_{i} = \ell(\beta_{0}, \; \boldsymbol{\beta}\;) = \ell[(1 - \mathbf{u}^{2} + \mathbf{u}\bullet\beta), - \mathbf{w}] \tag{5.2}$$

$$\beta^{i}\beta_{i} \;= \ell^{-2} = (1 - \mathbf{w}^{2}) \tag{5.3}$$

$$\nu^{i}\nu_{i} \;= \ell^{2}\, \beta^{i}\beta_{i} = 1 \tag{5.4}$$

*a. Symbols and terminologies for the null limits of* **u**, β, *and* **w**.

For $\mathbf{u} \to \mathbf{0}$, $\mathbf{w} \to \beta$ and $\ell \to 1/[1 - \beta^2]^{1/2} \equiv \gamma$. The environment is the undisturbed 3-space-energy continuum.

For $\beta \to \mathbf{0}$, $\mathbf{w} \to -\mathbf{u}$ and $\ell \to 1/[1 - \mathbf{u}^2]^{1/2} \equiv \Gamma$. The $p^\pi$ particle is immersed in the potentials $\Phi^i(x^i)$ while remaining at rest relative to Σ.

If $\mathbf{w} \to \mathbf{0}$ because $\beta = \mathbf{u} \to \mathbf{0}$: $\ell \to 1$ The $p^\pi$ particle is in its 3-space-energy *ground state* of *absolute rest in* ${}^0\mathcal{E}_m$.

If $\mathbf{w} \to \mathbf{0}$ because $\beta = -\mathbf{u}$: $\ell \to 1$ The $p^\pi$ particle is immersed in the potentials $\Phi^i(x^i)$ but is in a state of *convective free fall* or *relative absolute rest.*

2. The EM energy-momentum 4-vector $p^i$.

Converting momentum units to energy-units via an implicit factor of lightspeed, and defining $E^0 = m^0c^2$, the EM energy-momentum 4–vector has the following alternative forms:

$$p^i = (p^0, \mathbf{p}) = E^0\nu^i = (\ell E^0, \ell E^0\beta) = (E, E\beta) = E\beta^i. \tag{5.5}$$

For $\mathbf{u} = \mathbf{0}$, $\ell \to \gamma$, and $p^i$ attains the special relativistic form. Hence, with $p^\pi$ momentum conserved in the form prescribed by $p^i$, and with $\mathbf{F} = d\mathbf{p}/dx^0$ as an accurate description of the Newtonian force, we see that the equivalence of *mass and energy*, and the variation of *mass-energy* with $\delta\mathcal{E}_m$ propagation velocity, as previously deduced via GFT1 and described by

$$\underline{\underline{w}}d^4 \quad = \quad \underline{\underline{w}}d^5$$

$$mc^2/m^0c^2 = E/E^0 = \ell = 1/[1 - \mathbf{w}^2]^{1/2}, \tag{5.6}$$

can also be regarded as an immediate consequences of the expanded dynamics and the larger $\in_m$ physics underlying EM gravity that results from Maxwell's gravity theorem.

3. The EM Hamiltonian-momentum 4-vector $p_i$.

From Eqs. (4.14,15) I derive the following structures and interpretations of the Hamiltonian-momentum 4-vector:

$$p_i = g_{ik}p^k = (p_0, \vec{\mathbf{p}}) = E^0\nu_i = E\beta_i = [E(1 + \mathbf{u}\bullet\mathbf{w}), -\mathbf{w}E] = (H, -\mathbf{w}E), \text{ where} \tag{5.7}$$

$$H = E\beta_0 = E(1 - \mathbf{u}^2 + \mathbf{u}\bullet\beta) = (E - \vec{\mathbf{p}}\bullet\mathbf{u}) = [E(1 - \mathbf{u}^2) + \mathbf{p}\bullet\mathbf{u}], \tag{5.8}$$

describes the Hamiltonian of a $p^\pi$ particle in the $g_{ik}$ potentials of EM gravity.

*a. Terminologies for the null limits of* **u**, β, and **w**.

<u>The total energy of a $p^\pi$ particle is:</u>

| | | |
|---|---|---|
| For $\mathbf{u} = \mathbf{0}$, $H(u = 0, \beta = \mathbf{w}) = \gamma E^0 = E^0/[1 - \beta^2]^{1/2}$. | Above its ground state (rest) value. | (5.9) |
| For $\beta = \mathbf{0}$, $H(\beta = 0, \mathbf{w} = -\mathbf{u}) = E^0/\Gamma = (1 - \mathbf{u}^2)E$. | Below its ground-state (rest) value. | (5.10) |
| For $\mathbf{w} = \mathbf{0}$, $H(\beta = \mathbf{u}) = E^0$ (i.e., abs. or rel. free fall). | Equal to its ground state (rest) value. | (5.11) |

The phrase *at rest in EM gravity* will always be taken to mean *in the sense that* $\beta = \mathbf{0}$, and thus, $\mathbf{w} = -\,\mathbf{u}$. Hence, if a $p^{\pi}$ particle is at rest in EM gravity, it would take $(\Gamma - 1)H = E^0(1 - \Gamma^{-1})$ joules of work to pull it free of gravity and thus *up* to its ground-state energy $E^0$.

*b. The EM energy-Hamiltonian momentum scalar.*

The 4-scalar associated with $p^{\pi}$ energy and momentum in a $\Phi^i$ field has the following forms:

$$p^i p_i = [\,(p^0)^2 - \vec{\mathbf{p}}^{\,2}\,] = [\,E^2 - \vec{\mathbf{p}}^{\,2}\,] = (\,EH + \mathbf{p}\bullet\vec{\mathbf{p}}\,) = (E^0)^2. \tag{5.12}$$

B. The Hamiltonian, Lagrangian, and Action functions, and the Hamilton Jacobi Equations of a $p^{\pi}$ particle in the $\Phi^i$ potentials of EM gravity.

The following information may be of some use to theorists interested in formulating the *quantum characteristics* of EM gravity.

1. Consistent expressions and relations for Action.

As with $p^{\pi}$ momentum, I here use a lightspeed scaled action $S \equiv cS$. Hence, $p^i$, $p_i$, H, and the Lagrangian L, will consistently denote *mass-energy* while $S$ denotes *mass-energy-length* or *mass-energy-distance*. From the perspective of $\Sigma$ the quantities $S$, L, ds, and $p_i dx^i$ are interrelated as follows:

$$S = \int dS \equiv \int L dx^0 \qquad = -\int E^0 ds \quad = -\int p_i v^i ds = -\int p_i dx^i = -\int [p_0 dx^0 + \vec{\mathbf{p}}\bullet d\mathbf{r}], \tag{5.13a}$$

$$= -\int (E^0/\ell) dx^0 \;= -\int [p_0 + \vec{\mathbf{p}}\bullet \beta] dx^0 \;= -\int (p_i \beta^i) dx^0 = -E^0 \int d\underline{x}^0, \tag{5.13b}$$

$$= \int (\partial_i S) dx^i \quad = \quad \int [\,(\partial_0 S) dx^0 + (\nabla S)\bullet d\mathbf{r}] = \; -\int p_i dx^i. \tag{5.13c}$$

Equations (5.13) contain only three distinct integrands: A self-definition of differential *Action,* d$S$. The historic representation of d$S$ as $Ldx^0$. And five mathematically equivalent expressions of the *differential Action* 4-scalar, $dS = -\,p_i dx^i$, which is a (1 + 3) tensor generalization of the historically earliest Action integrand, $m^0 c\beta\bullet d\mathbf{r}$, associated with the principle of least action.

2. Lagrangian and Hamiltonian relations unique to EM gravity.

From Eqs. (5.13), I obtain the following equivalent expressions for the Lagrangian of a $p^{\pi}$ particle in the $g_{ik}$ potentials of EM gravity at $x^i$ of $\Sigma^d$:

$$L = -\,E^0\,[1 - (\beta - \mathbf{u})^2\,]^{1/2} = -\,(p_0 - \mathfrak{p}\bullet\beta) = -(H - \mathfrak{p}\bullet\beta) = -\,p_i\beta^i = -\,p^i\beta_i, \tag{5.14}$$

where I have introduced the symbol, $\mathfrak{p} \equiv -\vec{\mathbf{p}}$ , to denote Hamilton's *generalized momentum* (scaled by c to convert it to energy units). Hamilton's generalized momentum then has the following definition and meaning for EM gravity:

$$\mathfrak{p} = \partial L/\partial\beta \;=\; \mathbf{w}E \;=\; -\;\vec{\mathbf{p}} \;=\; \mathbf{p} - \mathbf{u}E \;=\; \nabla S, \tag{5.15}$$

with,

$$p_0 = -\,\partial S/\partial x^0\,, \tag{5.16a}$$

$$\vec{\mathbf{p}} \;=\; -\,\nabla S. \tag{5.16b}$$

## 3. Hamilton's function and the Hamiltonian for EM gravity.

Eq. (5.15) shows that $\mathfrak{p}$ is generally different from both $\vec{\mathbf{p}}$ and $\mathbf{p}$. These definitions and relations, stemming directly from Eqs. (5.13), are consistent with Hamilton's function:

$$\mathrm{H}(\mathfrak{p},\mathbf{u}) = \mathfrak{p}\bullet\beta - \mathrm{L}(\beta,\mathbf{u}), \tag{5.17}$$

$$= \mathrm{E}^0[(1-\mathbf{u}^2) + \mathbf{u}\bullet\beta]/(1-\mathbf{w}^2), \tag{5.18}$$

for converting $\mathrm{L}(\beta,\mathbf{u})$ into $\mathrm{H}(\mathfrak{p}, \mathbf{u})$, and for generating the partial differential *Hamiltonian-Jacobi* [Action] equation, symbolically denoted by

$$\mathrm{H}(\nabla S, \mathrm{x}^i) + \partial S/\partial \mathrm{x}^0 = 0. \tag{5.19}$$

Comparing (5.19) to (5.16a), we recover the well-known fact that $\mathrm{p}_0 = \mathrm{H}$. That is, the temporal component of the covariant 4-energy-momentum vector contains the total energy of a $\mathrm{p}^\pi$ particle. Thus it is appropriate to call $\mathrm{p}_i$ the *Hamiltonian-momentum 4-vector*. We then both confirm and derive that the equations

$$\mathrm{H}(\mathfrak{p}, \mathbf{u}) = \mathrm{p}_0 = [(\mathfrak{p})^2 + (\mathrm{E}^0)^2\,]^{1/2} + \mathbf{u}\bullet\mathfrak{p}\,, \tag{5.20a}$$

$$= \mathrm{E} + \mathbf{u}\bullet\mathfrak{p}, \tag{5.20b}$$

describe the standard and the non standard *Hamiltonians* for a $\mathrm{p}^\pi$ particle in the field of EM gravity at $\mathrm{x}^i$ of $\Sigma^d$, with the non standard expression, (5.20b), illuminating the fact that the difference between H and E is driven by $\mathbf{u}\bullet\mathfrak{p}$.

## 4. The Hamilton-Jacobi equation for $\mathrm{p}^\pi$ and $\mathrm{p}^0$ particles in EM gravity.

Using $\partial_i \equiv \partial/\partial \mathrm{x}^i$, the *Hamilton-Jacobi* equation takes the explicit forms:

$$[\,(\nabla S)^2 + \mathrm{g}^{ik}(\partial_i S)\,(\partial_k S)\,]^{1/2} + \mathbf{u}\bullet\nabla S + \partial_0 S = 0, \tag{5.21}$$

$$[\mathbf{u}\bullet\nabla S + \partial_0 S\,]^2 - (\nabla S)^2 - \mathrm{g}^{ik}(\partial_i S)\,(\partial_k S) = 0, \tag{5.22}$$

where,
$$\mathrm{g}^{ik}(\partial_i S)\,(\partial_k S) = \mathrm{g}^{ik}\mathrm{p}_i\mathrm{p}_k = \mathrm{p}^i\mathrm{p}_i = (\mathrm{E}^0)^2. \tag{5.23}$$

These equations govern the temporal evolution of the Action of a p particle in the gauge field of Einstein-Maxwell gravity at $\mathrm{x}^i$ of $\Sigma^d$. For a $\mathrm{p}^0$ particle, $\mathrm{E}^0 = 0$, and they reduce to:

$$|\nabla S| + \mathbf{u}\bullet(\nabla S) + \partial S/\partial \mathrm{x}^0 = 0. \tag{5.24}$$

5. Hamilton's canonical equation f motion for $\beta(p)$ in the field of EM gravity.

Using Hamilton's canonical equation of motion for the 3-velocity of a p particle,

$$\beta(p) = \partial H/\partial \mathfrak{p}, \tag{5.25}$$

and Eqs. (5.20), I obtain:

$$\beta(p) = \partial H/\partial \mathfrak{p} = \mathfrak{p}/[(\mathfrak{p})^2 + (E^0)^2]^{1/2} + \mathbf{u},$$

$$= \mathfrak{p}/E + \mathbf{u}, \tag{5.26}$$

$$= \mathbf{w} + \mathbf{u} \text{ for } E^0 > 0, \tag{2.3}$$

$$= \mathbf{w}^0 + \mathbf{u} \text{ for } E^0 = 0, \tag{2.3}$$

thereby recovering the cornerstone GRV equation (2.3) of hyper energy theory.

6. On the small w convergence of H to its pre-relativistic form; $T + V_H$.

Expanding (5.8) and retaining only second order terms in **u** and $\beta$, we obtain $H = T + V_H$ where,

$$T = \tfrac{1}{2}E^0 \beta^2, \tag{5.27}$$

$$V_H = -E^0(\tfrac{1}{2}\mathbf{u}^2) + E^0, \tag{5.28a}$$

$$\equiv -E^0\Phi_m + E^0. \tag{5.28b}$$

Since the constant $E^0$ term in $V_H$ does not affect interaction forces, we see that in this small w approximation the Hamiltonian for a $p^\pi$ particle of rest energy $E^0$ in an EM gravity field agrees with the pre relativistic formulation of H, provided that $(-\tfrac{1}{2}\mathbf{u}^2)E^0$ is the hyper energy explanation for the Newtonian potential, $\Phi_n$, in accordance with the previous deduction

$$\Phi_m = \tfrac{1}{2}\mathbf{u}^2 = -\Phi_n/c^2, \tag{3.80a}$$

from Maxwell's Gravity theorem.

Since this result follows simply from $ds = dx^0/\ell$ and the identification of $p_0$ as H, it offers yet another example of the extreme effectiveness of the (1 + 3) tensor calculus.

8. On the small w convergence of L to its pre relativistic form; $T - V_L$.

Expanding (5.14) and retaining only second order terms in **u** and $\beta$, we obtain $L = T - V_L$, where

$$T = \tfrac{1}{2}E^0 \beta^2, \tag{5.29}$$

$$V_L - E^0 = -E^0(\Phi_m - \mathbf{u}\bullet\beta), \tag{5.30}$$

$$= -E^0\Phi_i\beta^i, \tag{5.31}$$

where,

$$\Phi_i = (\Phi_m, \vec{\Phi}) = (\Phi_m, -\mathbf{u}). \tag{5.32}$$

Since the constant $E^0$ term in $V_L$ does not affect the gravitational interaction, and since it is understood that $-E^0\Phi_m$ is the stationary gravitational mass defect, we see that, in this small w approximation, the Lagrangian for $E^0$ in an EM gravity field contains the pre relativistic formulation plus a field-GRV coupling term which gives the gravitational interaction potential, $V_L$, the same structure as the electromagnetic interaction potential, $L_e$, in a Lagrangian for a charge e in an electromagnetic field possessing a covariant 4-potential, $\varphi_i = -(\varphi_e, -c\mathbf{A})$. Namely,

$$L_e = -e\varphi_i\beta^i = -e(\varphi_e, -c\mathbf{A}\bullet\beta). \tag{5.33}$$

Thus, although $\Phi_i = (\Phi_m, -\mathbf{u})$ is not a covariant 4-vector, it is nevertheless functionally identical to the covariant electromagnetic 4-potential $\varphi_i = (\varphi_e, -c\mathbf{A})$, and as such it provide additional assurance that EM gravity can be readily quantized—to predict the kinds of $\delta\mathcal{E}_m$ particles that can be precipitated specifically by collisionally unleashed gravitational mass-energy, as apposed to collisionally unleashed kinetic mass-energy.

C. The second order dynamics of a free $p^\pi$ particle in EM gravity.

1. The contravariant power and force equations.

With $g_{ik}(u^\alpha(x^i))$ and $g^{ik}(u^\alpha(x^i))$ defined by Eqs. (4.13), and $p^i(x^i)$ defined by Eqs. (5.5-6), we now utilize the well known (1 + 3) tensor equations:

$$Dp^i/ds = dp^i/ds + \Gamma^i{}_{jk}\, p^j\nu^k = \mathbf{F}^{\mathbf{i}}(x^i), \tag{5.34}$$

where,

$$\Gamma^i{}_{jk}(x^i) = \Gamma^i{}_{kj} = g^{im}[\, \partial g_{jm}/\partial x^k + \partial g_{mk}/\partial x^j - \partial g_{jk}/\partial x^m\, ], \tag{5.35}$$

to describe, at $x^i$ of $\Sigma^d$, the contravariant dynamics of a $p^\pi$ particle in a general $g_{ik}(u^\alpha(x^i))$ field of EM gravity in response to a *non-gravitational* 4-force, $\mathbf{F}^{\mathbf{i}}$, subject to the tentative *free field-particle* restriction defined by $\mathbf{F}^{\mathbf{i}} = 0$. The contravariant equation of motion for a free $p^\pi$ particle in an EM gravity field is then:

$$dp^i/ds = -\Gamma^i{}_{jk}\, p^j\nu^k. \tag{5.36}$$

*a. The 4-rate of change of contravariant $p^\pi$ energy, $dp^0/ds$.*

Using the components of $\Gamma^0{}_{jk}(x^i)$ listed in Appendix A, and $p^0 = E$, we obtain:

$$dp^0/ds = \ell dp^0/dx^0 = -\Gamma^0{}_{jk}p^j\nu^k = E\ell\psi, \tag{5.37}$$

where,

$$\psi = -\mathbf{w}\bullet[(\mathbf{w}\bullet\nabla)\mathbf{u}]. \tag{5.38}$$

Hence

$$d[\ln(E/E^0)]/dx^0 = [d(½\mathbf{w}^2)/dx^0]/(1 - \mathbf{w}^2) = -\mathbf{w}\bullet[(\mathbf{w}\bullet\nabla)\mathbf{u}] = \psi. \tag{5.39}$$

*b. The 4-rate of change of contravariant $p^\pi$ momentum, d**p**/ds.*

Using the components of $\Gamma^\alpha{}_{jk}(x^i)$ listed in Appendix A, $\Gamma_{jk}(x^i) = \mathbf{e}_\alpha\Gamma^\alpha{}_{jk}(x^i)$, and $\mathbf{p} = \beta E$, we obtain:

$$d\mathbf{p}/ds = \ell d\mathbf{p}/dx^0 = -\,\Gamma_{jk}(x^i)p^j v^k, \tag{5.40}$$

$$d\mathbf{p}/dx^0 = E\,[\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \beta\times(\nabla\times\mathbf{u}) + \psi\mathbf{u}\,]. \tag{5.41}$$

And inserting Eq. (5.39) into Eq. (5.41), we then obtain the Σ–acceleration of a free $p^\pi$ particle in a general $g_{ik}(u^\alpha(x^i))$ field of EM gravity as:

$$d\beta/dx^0 = d^2\mathbf{r}/(dx^0)^2 = \mathbf{a}(x^i) = [\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \beta\times(\nabla\times\mathbf{u}) - \psi\mathbf{w}\,], \tag{5.42}$$

$$= [\,d\mathbf{u}/dx^0 - \mathbf{w}\times(\nabla\times\mathbf{u}) - \psi\mathbf{w}\,], \tag{5.43}$$

where

$$d\mathbf{u}/dx^0 = \partial\mathbf{u}/\partial x^0 + (\beta\bullet\nabla)\mathbf{u}, \tag{5.44a}$$
$$= \partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \mathbf{u}\times(\nabla\times\mathbf{u}). \tag{5.44b}$$

From (5.43) it is clear that, with respect to Σ, the acceleration of a $p^\pi$ particle in *free fall* ($\mathbf{w} = 0$) reduces to the well-known expression (5.44) for the total 3-acceleration of a fluid medium. And from (5.38) and (5.42) we find that $p^\pi$ particle which is instantaneously at rest at $x^i$ of $\Sigma^d$ will initially accelerate at the rate

$$d\beta/dx^0 = d^2\mathbf{r}/(dx^0)^2 = \mathbf{a}(x^i) = [\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \mathbf{u}(\mathbf{u}\bullet\nabla\Phi_m)\,]. \tag{5.45}$$

*c. The remarkable similarity of electrodynamic and gravodynamic forces.*

Consider the pair of equations:

$$d(\ell E^0\beta)/dx^0 = \mathbf{f}_{Em} = E\,[\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \beta\times(\nabla\times\mathbf{u}) + (\psi)\mathbf{u}\,]. \tag{5.41}$$

$$d(\gamma E^0\beta)/dx^0 = f_L = cQ_-[\,\partial\mathbf{A}/\partial x^0 + \nabla(\varphi/c) - \beta\times(\nabla\times\mathbf{A}) + (0)\mathbf{A}\,]. \tag{5.46}$$

The first equation describes the generally covariant *EM gravity force* ($\mathbf{f}_{Em}$) on a $p^\pi$ particle with rest energy $E^0$ which is *moving* with a GRV, $\beta = \mathbf{u} + \mathbf{w}$, through the *acceleration-field potentials* $g_{ik}(u^\alpha)$ of EM gravity at $x^i$ of $\Sigma^d$. While (5.46) describes the covariant *Lorentz force* ($\mathbf{f}_L$) on a $p^\pi$ particle with rest energy $E^0$ and negative electric charge $Q_-$ which is *propagating* with GRV, $\beta = \mathbf{w}$, through the electromagnetic potentials, $\varphi^i_e(x^i) = (\varphi_e, \mathbf{A}^*)$, at $x^i$ of $\Sigma^d$ where $g_{ik} = G_{ik}$ = diagonal (1, −1, −1, −1).

Since electromagnetism is readily quantizable, this remarkable similarity provides further assurance of the quantizability of EM gravity.

2. The covariant power and force equations.

With $g_{ik}(u^{\alpha}(x^{i}))$ defined by Eqs. (4.13), and $p_i(x^i)$ defined by Eqs. (5.7-8), we now utilize the well-known (1 + 3) tensor equations:

$$Dp_i/ds = dp_i/ds - ½(\partial g_{jk}/\partial x^i)\, p^j \nu^k = \mathbf{F_i}(x^i) = 0, \tag{5.47}$$

to describe, at $x^i$ of $\Sigma^d$, the covariant dynamics of a $p^{\pi}$ particle in a general $g_{ik}(u^{\alpha}(x^{i}))$ field of EM gravity in response to a *non-gravitational* 4-force, $\mathbf{F_i}$, subject to the tentative *free field-particle* restriction defined by $\mathbf{F_i} = 0$. The covariant equation of motion for a free $p^{\pi}$ particle in an EM gravity field is then:

$$dp_i/ds = ½(\partial g_{jk}/\partial x^i)\, p^j \nu^k. \tag{5.48}$$

*a. The 4-rate of change of total $p^{\pi}$ energy, $dp_0/ds$.*

Using $p_0 = H$, and $\partial_0 = \partial/\partial x^0$, the 4-rate of change of the total energy of a free $p^{\pi}$ particle in an EM gravity field is described by

$$dp_0/ds = dH/ds = \ell dH/dx^0 = ½\, \partial_0[g_{jk}(u^{\alpha})]\, p^j \nu^k = ½\ell E\, \partial_0[g_{jk}(u^{\alpha})]\, \beta^j \beta^k, \tag{5.49a}$$

$$= \ell E\, \mathbf{w}\bullet(\partial \mathbf{u}/\partial x^0), \tag{5.49b}$$

or,

$$dH/dx^0(x^i) = E^0\, \mathbf{w}\bullet(\partial \mathbf{u}/\partial x^0)/[1 - \mathbf{w}^2]^{½}, \tag{5.49c}$$

$$= - E^0\, \Gamma(\partial \Phi_m/\partial x^0), \text{ (for } \beta = \mathbf{0}). \tag{5.49d}$$

*b. The 4-rate of change of covariant $p^{\pi}$ momentum,* $d\vec{\mathbf{p}}/ds$.

The 4-rate of change of the covariant component of $p^{\pi}$ momentum is then given by

$$d\vec{\mathbf{p}}/d\mathbf{s} = \ell dp_{\gamma}/dx^0 = ½(\partial g_{jk}/\partial x^{\gamma})\, p^j \nu^k = ½\ell E\, \partial_{\gamma}[g_{jk}(u^{\alpha})]\, \beta^j \beta^k, \tag{5.50a}$$

$$= E\,[\, - \nabla \Phi_m + \beta \times (\nabla \times \mathbf{u}) + \beta \bullet \nabla)\mathbf{u}\,]. \tag{5.50b}$$

D. The general (1 + 3) EM **w**-deformations of (1 + 3) tensor quantities.

As in IV.E, we let $\underline{\underline{v}}^i = \underline{\underline{\ell}}\underline{\underline{\beta}}^i$ be the 4–GRV of both $\underline{O}$ and $\underline{\mathbf{r}}$, so that the reference point $`\underline{\mathbf{r}}$ and a $\delta\mathcal{E}_m$ particle at $x^i$ of $\Sigma^d$ are embedded in the same $\in_m$ flow field, $\underline{\underline{\mathbf{u}}} = \mathbf{u}$. Hence, $\underline{\underline{\mathbf{w}}}$ and $\mathbf{w}$ are thus consistently defined by $\underline{\underline{\mathbf{w}}} = \underline{\underline{\beta}} - \mathbf{u}$, and $\mathbf{w} = \beta - \mathbf{u}$.

The temporal component of a general Lorentz transformation,

$$d\underline{x}^0 = \underline{\underline{v}}_i dx^i = \underline{\underline{\boldsymbol{\ell}}}[(1 + \mathbf{u}\bullet\underline{\underline{\mathbf{w}}}), -\underline{\underline{\mathbf{w}}}]\bullet(dx^0, d\mathbf{r}), \tag{4.17}$$

$$= \underline{\underline{\ell}}(1 - \mathbf{w}\bullet\underline{\underline{\mathbf{w}}})dx^0, \tag{4.17}$$

may then be regarded as the temporal component of the (1 + 3) tensor equation,

$$d\underline{x}^i(\underline{\underline{\beta}}, \mathbf{u}, dx^k) = E^i{}_k(\underline{\underline{\beta}}, \mathbf{u})dx^k, \tag{5.51}$$

which describes the degree to which the numbers, $d\underline{x}^i$, that quantify contravariant tensor elements in $\underline{\Sigma}$ are unwittingly *inflated* or *deflated* relative to the $dx^k$, due to the fact that the relevant measurement standards in $\underline{\Sigma}$ are unknowingly w-deformed.

For example. Take the case of an oscillator which is at rest at $\underline{O}$ of $\underline{\Sigma}$ in the $\Gamma$-coupling domain. Then $d\underline{x}^0 = \Gamma(1 - \mathbf{u}^2)dx^0 = dx^0/\Gamma$ shows that the number obtained in $\underline{\Sigma}$ for the period of the oscillator is unwittingly *deflated,* due to $\underline{\underline{w}}d^2$.

It follows that the inverse equation

$$dx^i(\underline{\underline{\beta}}, \mathbf{u}, d\underline{x}^k) = D^i{}_k(\underline{\underline{\beta}}, \mathbf{u})d\underline{x}^k, \tag{5.52}$$

describes how the $d\underline{x}^i$ at $\underline{O}$ in $\underline{\Sigma}$—regarded as empirically quantified proper quantities—are gauge deformed by $\underline{\underline{\beta}}$ and/or $\mathbf{u}$ so as to preserve the Null-$\underline{\mathbf{p}}_m$ Axiom, thereby causing $\underline{\Sigma}^\ell$ to be empirically equivalent to $\Sigma^\ell$. Using the previous example, $dx^0 = \Gamma d\underline{x}^0$, or $T = \Gamma\, \underline{T}^0 = \Gamma\, T^0$, shows that, from the perspective of $\Sigma$, the period of an oscillator at rest in $\underline{\Sigma}$ is increased by the $\Gamma$-coupling in accordance with $\underline{\underline{w}}d^2$. And since $dx^i dx_i = d\underline{x}^k d\underline{x}_k = ds^2$ it follow that $[D] = [E]^{-1}$ and/or $D^i{}_m D^m{}_k = \delta^i{}_k$.

It follows that the $D^i{}_k$ contain $\underline{\underline{w}}d^{1-3}$ in a form that applies specifically to the elements of a contravariant 4-vector, for arbitrary $\underline{\underline{\mathbf{w}}}$ and $\mathbf{u}$. And it then follows that the $D^i{}_k$ deformations must apply equally well to *any contravariant vector*, $\underline{A}^i$. And since $\underline{A}^i\underline{A}_i = A^i A_i$ , we must conclude that whereas *contravariant* 4-vectors are **w**-deformed by

$$A^i(\underline{\underline{\beta}}, \mathbf{u}, \underline{A}^k) = D^i{}_k(\underline{\underline{\beta}}, \mathbf{u})\underline{A}^k, \tag{5.53a}$$

*covariant* 4-vectors are **w**-deformed by

$$A_i(\underline{\underline{\beta}}, \mathbf{u}, \underline{A}_k) = E^k{}_i(\underline{\underline{\beta}}, \mathbf{u})\underline{A}_k. \tag{5.54}$$

Given $E^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$ and $D^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$ one can then obtain the more complicated **w-**deformation of a (1 + 3) tensor of arbitrary mixed rank in accordance with the rule indicated by

$$T_{ik} = E^\ell{}_i E^m{}_k \underline{T}_{\ell m}\,, \tag{5.55}$$

$$T^{ik} = D^i{}_\ell D^k{}_m \underline{T}^{\ell m}\,. \tag{5.56}$$

And we use this rule to actually derive $E^i{}_k$ and $D^i{}_k$ in accordance with the following expanded form of the Null-$\underline{\mathbf{p}}_m$ Axiom.

1. The expanded Null-$\underline{\mathbf{p}}_m$ Axiom.

The expanded Null-$\underline{\mathbf{p}}_m$ Axiom states that:

The $g_{ik}(u^\alpha)$ at $\underline{x}^i$ of $\Sigma^d$ must be compatible with
$\underline{g}_{ik} = \underline{g}^{ik} \equiv G_{ik} = \text{diagonal}\ (1, -1,-1,-1)$ at $\underline{O}$ of $\underline{\Sigma}$.

The tensor $E^i{}_k$ (and thus $D^i{}_k$) can then be derived from the equations

$$g_{ik}(u^\alpha) = E^\ell{}_i E^m{}_k\, G_{\ell m}, \tag{5.57}$$

$$g^{ik}(u^\alpha) = D^\ell{}_i D^m{}_k\, G^{\ell m}, \tag{5.58}$$

by using equations (4.13) for $g_{ik}(u^\alpha)$ and $g^{ik}(u^\alpha)$ , and

$$E^0{}_i(\underline{\underline{\beta}}, \mathbf{u}) = \underline{\underline{v}}_i, \tag{5.59}$$

from (4.17) and (5.51). The derivations are carried out in Appendix B and the results are displayed as follows.

2. Contravariant 4-vectors are gauge deformed by:

$$D^i{}_k = \begin{array}{c c} & \begin{array}{cccc} j\to 0 & 1 & 2 & 3 \end{array} \\ i & \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \left[\begin{array}{cccc} \underline{\underline{\ell}} & \underline{\underline{\ell}}\,\underline{\underline{w}}_1 & \underline{\underline{\ell}}\,\underline{\underline{w}}_2 & \underline{\underline{\ell}}\,\underline{\underline{w}}_3 \\ \underline{\underline{\ell}}\,\underline{\underline{\beta}}_1 & 1+[g\underline{\underline{w}}_1+\underline{\underline{\ell}}u_1]\underline{\underline{w}}_1 & [g\underline{\underline{w}}_1+\underline{\underline{\ell}}u_1]\underline{\underline{w}}_2 & [g\underline{\underline{w}}_1+\underline{\underline{\ell}}u_1]\underline{\underline{w}}_3 \\ \underline{\underline{\ell}}\,\underline{\underline{\beta}}_2 & [g\underline{\underline{w}}_2+\underline{\underline{\ell}}u_2]\underline{\underline{w}}_1 & 1+[g\underline{\underline{w}}_2+\underline{\underline{\ell}}u_2]\underline{\underline{w}}_2 & [g\underline{\underline{w}}_2+\underline{\underline{\ell}}u_2]\underline{\underline{w}}_3 \\ \underline{\underline{\ell}}\,\underline{\underline{\beta}}_3 & [g\underline{\underline{w}}_3+\underline{\underline{\ell}}u_3]\underline{\underline{w}}_1 & [g\underline{\underline{w}}_3+\underline{\underline{\ell}}u_3]\underline{\underline{w}}_2 & 1+[g\underline{\underline{w}}_3+\underline{\underline{\ell}}u_3]\underline{\underline{w}}_3 \end{array}\right] \end{array}, \tag{5.60}$$

where

$$g = g(\underline{\underline{\ell}}) = \underline{\underline{\ell}}^2/(1+\underline{\underline{\ell}}) = \underline{\underline{\ell}}\ f(\underline{\underline{\ell}}), \tag{5.61}$$

*a. The elements and limits of* $D^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

$$D^0{}_k = \underline{\underline{\ell}}(1, \underline{\underline{w}}^\alpha), \tag{5.62a}$$

$$= \underline{\underline{\gamma}}(1, \underline{\underline{\beta}}^\alpha) \quad \text{in the } \gamma\text{-coupling domain,} \tag{5.62b}$$

$$= \Gamma(1, -u^\alpha) \text{ in the } \Gamma\text{-coupling domain.} \tag{5.62c}$$

$$D^\alpha{}_0 = \underline{\underline{\ell}}\,\underline{\underline{\beta}}^\alpha \neq D^0{}_\alpha \text{ (unless } \mathbf{u} = \mathbf{0}). \tag{5.63a}$$

$$= \underline{\underline{\gamma\beta}}^\alpha \quad \text{in the } \gamma\text{-coupling domain,} \tag{5.63b}$$

$$= 0 \quad \text{in the } \Gamma\text{-coupling domain.} \tag{5.63c}$$

$$D^\alpha{}_\upsilon = \delta^\alpha{}_\upsilon + [g\,\underline{\underline{w}}^\alpha + \underline{\underline{\ell}}\,u^\alpha]\underline{\underline{w}}^\upsilon, \tag{5.64a}$$

$$= \delta^\alpha{}_\upsilon + g(\underline{\underline{\gamma}})\underline{\underline{\beta}}^\alpha\underline{\underline{\beta}}^\upsilon \text{ in the } \gamma\text{-coupling domain,} \tag{5.64b}$$

$$= \delta^\alpha{}_\upsilon - f(\Gamma)u^\alpha u^\upsilon \text{ in the } \Gamma\text{-coupling domain (see (5.61) for f).} \tag{5.64c}$$

*b. The γ-coupling and Γ-coupling domain limits of* $D^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

In the γ-coupling domain with:

$$g = g(\underline{\underline{\gamma}}) = \underline{\underline{\gamma}}^2/(1 + \underline{\underline{\gamma}}) \qquad \text{and} \qquad \underline{\underline{\beta}}^\alpha = \underline{\underline{\beta}}^1 = \underline{\underline{\beta}}.$$

$$D^i{}_k = \begin{array}{cc} \mathbf{i}\ \ \mathbf{j}\to & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \underline{\underline{\gamma}} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 \\ \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & 1 + g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_3 \\ \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_1 & 1 + g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_3 \\ \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_2 & 1 + g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_3 \end{bmatrix} \end{array} \rightarrow \begin{array}{cc} \mathbf{i}\ \ \mathbf{j}\to & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \underline{\underline{\gamma}} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}} & 0 & 0 \\ \underline{\underline{\gamma}}\,\underline{\underline{\beta}} & \underline{\underline{\gamma}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \tag{5.65}$$

And in the Γ-coupling domain with:

$$f = f(\Gamma) = \Gamma/(1 + \Gamma) \qquad \text{and} \qquad \text{and } u^\alpha = u^1 = u.$$

$$D^i{}_k = \begin{array}{cc} \mathbf{i}\ \ \mathbf{j}\to & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \Gamma & -\Gamma u_1 & -\Gamma u_2 & -\Gamma u_3 \\ 0 & 1 - fu_1^2 & -fu_1u_2 & -fu_1u_3 \\ 0 & -fu_2u_1 & 1 - fu_2^2 & -fu_2u_3 \\ 0 & -fu_3u_1 & -fu_3u_2 & 1 - fu_3^2 \end{bmatrix} \end{array} \rightarrow \begin{array}{cc} \mathbf{i}\ \ \mathbf{j}\to & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \Gamma & -\Gamma u & 0 & 0 \\ 0 & \Gamma^{-1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \tag{5.66}$$

3. Covariant 4-vectors are then **w**-gauge deformed by:

$$E^i{}_k = \begin{array}{cc} \mathbf{i}\ \ \mathbf{j}\to & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \underline{\underline{\boldsymbol{\ell}}}(1 + \mathbf{u}\bullet\underline{\underline{\mathbf{w}}}) & -\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_1 & -\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_2 & -\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_3 \\ -(\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_1 + G_1) & 1 + g\,\underline{\underline{w}}_1^2 & g\,\underline{\underline{w}}_1\,\underline{\underline{w}}_2 & g\,\underline{\underline{w}}_1\,\underline{\underline{w}}_3 \\ -(\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_2 + G_2) & g\,\underline{\underline{w}}_2\,\underline{\underline{w}}_1 & 1 + g\,\underline{\underline{w}}_2^2 & g\,\underline{\underline{w}}_2\,\underline{\underline{w}}_3 \\ -(\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_3 + G_3) & g\,\underline{\underline{w}}_3\,\underline{\underline{w}}_1 & g\,\underline{\underline{w}}_3\,\underline{\underline{w}}_2 & 1 + g\,\underline{\underline{w}}_3^2 \end{bmatrix} \end{array}, \tag{5.67}$$

where

$$g = g(\underline{\underline{\ell}}) = \underline{\underline{\ell}}^2/(1 + \underline{\underline{\ell}}) = \underline{\underline{\ell}}\, f(\underline{\underline{\ell}}). \tag{5.68}$$

$$\mathbf{G} = \{\, g[\underline{\underline{\mathbf{w}}}\times(\underline{\underline{\mathbf{w}}}\times\mathbf{u}) + \underline{\underline{\ell}}\mathbf{u} \,\}, \tag{5.69a}$$

$$= \{\, g(\mathbf{u}\bullet\underline{\underline{\mathbf{w}}})\underline{\underline{\mathbf{w}}} + \mathbf{u} \,\}. \tag{5.69b}$$

*a. The elements and limits of* $E^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

$$E^0{}_k = \underline{\underline{\ell}}[(1 + \mathbf{u}\bullet\underline{\underline{\mathbf{w}}}), -\underline{\underline{w}}^\alpha] \tag{5.70a}$$

$$= \underline{\underline{\gamma}}(1, -\underline{\underline{\beta}}^\alpha) \text{ in the } \gamma\text{-coupling domain,} \tag{5.70b}$$

$$= \Gamma[(1 - \mathbf{u}^2), u^\alpha) \text{ in the } \Gamma\text{-coupling domain.} \tag{5.70c}$$

$$E^\alpha{}_0 = -\{\underline{\underline{\ell}}\, \underline{\underline{w}}^\alpha + g(\mathbf{u}\bullet\underline{\underline{\mathbf{w}}})\underline{\underline{w}}^\alpha + u^\alpha\} \neq E^0{}_\alpha \text{ (unless } \mathbf{G} = \mathbf{0}). \tag{5.71a}$$

$$= -\underline{\underline{\gamma}}\underline{\underline{\beta}}^\alpha \text{ in the } \gamma\text{-coupling domain,} \tag{5.71b}$$

$$= 0 \quad \text{in the } \Gamma\text{-coupling domain.} \tag{5.71c}$$

$$E^\alpha{}_\upsilon = \delta^\alpha{}_\upsilon + g\, \underline{\underline{w}}^\alpha \underline{\underline{w}}^\upsilon, \tag{5.72a}$$

$$= \delta^\alpha{}_\upsilon + g(\underline{\underline{\gamma}})\underline{\underline{\beta}}^\alpha\underline{\underline{\beta}}^\upsilon \text{ in the } \gamma\text{-coupling domain,} \tag{5.72b}$$

$$= \delta^\alpha{}_\upsilon + g(\Gamma)u^\alpha u^\upsilon \text{ in the } \Gamma\text{-coupling domain.} \tag{5.72c}$$

*b. The* $\gamma$*-coupling and* $\Gamma$*-coupling domain limits of* $E^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

In the $\gamma$-coupling domain with:

$$g = g(\underline{\underline{\gamma}}) = \underline{\underline{\gamma}}^2/(1 + \underline{\underline{\gamma}}) \qquad \text{and} \qquad \underline{\underline{\beta}}^\alpha = \underline{\underline{\beta}}^1 = \underline{\underline{\beta}}.$$

$$E^i{}_k = \begin{array}{c|c} & \mathbf{j} \rightarrow \\ \mathbf{i} & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \underline{\underline{\gamma}} & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 \\ -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & 1 + g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_3 \\ -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_1 & 1 + g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_3 \\ -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_2 & 1 + g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_3 \end{bmatrix} \end{array}, \rightarrow \begin{array}{c|c} & \mathbf{j} \rightarrow \\ \mathbf{i} & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \underline{\underline{\gamma}} & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}} & 0 & 0 \\ -\underline{\underline{\gamma}}\,\underline{\underline{\beta}} & \underline{\underline{\gamma}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \tag{5.73}$$

And in the $\Gamma$-coupling domain with:

$$g = g(\Gamma) = \Gamma^2/(1 + \Gamma) \qquad \text{and} \qquad \text{and } u^\alpha = u^1 = u.$$

$$E^i{}_k = \begin{array}{c|c} & \mathbf{j} \rightarrow \\ \mathbf{i} & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \Gamma^{-1} & \Gamma u_1 & \Gamma u_2 & \Gamma u_3 \\ 0 & 1 + gu_1{}^2 & gu_1u_2 & gu_1u_3 \\ 0 & gu_2u_1 & 1 + gu_2{}^2 & gu_2u_3 \\ 0 & gu_3u_1 & gu_3u_2 & 1 + gu_3{}^2 \end{bmatrix} \end{array} \rightarrow \begin{array}{c|c} & \mathbf{j} \rightarrow \\ \mathbf{i} & \begin{array}{cccc} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{array}{c} \mathbf{0} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} & \begin{bmatrix} \Gamma^{-1} & \Gamma u & 0 & 0 \\ 0 & \Gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}. \tag{5.74}$$

E. A classical solution of the photon gravitational-mass/redshift problem.

*a. The problem.*

In accordance with both **w**-gauge theory, EM gravity and the perspective of Σ, any oscillator at a fixed position in a steady gravitational field will run slow by the factor $1/\ell \rightarrow 1/\Gamma = g_{00}{}^{½} = [1 - \mathbf{u}^2]^{½}$, causing its Σ−frequency to be reduced to $\upsilon = \upsilon_0/\Gamma$, where $\upsilon_0$ is its standard, proper, rest, or w = 0, frequency. This Γ−expression of $\underline{\underline{w}}d^2$, and the γ−expression of $\underline{\underline{w}}d^2$ as well, were experimentally confirmed in early 1970 by means of Cesium-beam clocks.[30] And since the Σ−frequency of any kind of radiation must remain constant as it propagates throughout a steady medium,[31] the Cesium-beam clock experiments served to prove that the Γ−expression of $\underline{\underline{w}}d^2$ is sufficient to account for the gravitational redshift—solely in terms of the spatial constancy of the reduced frequency of the emitted photon, $\upsilon = \upsilon_e = \upsilon_0/\Gamma_e$.

Also, in accordance with the fundamental equation, $\lambda = h/p$, of quantum mechanics and the equally fundamental equation, $E = mc^2$, of special relativity theory and **w**-gauge theory, a photon has a purely kinetic energy and mass which are described in the γ−coupling domain by $E = h\upsilon = mc^2$.

Hence, the *photon gravitational-mass/redshift problem* is stated as follows:

> Given that the kinetic mass-energy of a photon is accurately expressed by $E = h\upsilon$, and that the Σ−frequency of a photon remains constant throughout a steady gravity field, how can we explain the photon's gravitational coupling, and thus the mass-energy that a photon *looses* in the course of propagating gravitationally uphill?

*b. A solution.*

The energy of a photon at a point where the gravitational potential is φ can be expressed equally well as $E_\varphi = hc/\lambda_\varphi$. And $\lambda_\varphi$ is of course free to vary provided that its variation is linked to a variation in its intrinsic propagation speed, $w^0_\varphi$. We are thus at liberty to say, firstly, that the energy of a photon E is defined more generally by

$$E_\varphi = hc/\lambda_\varphi\,. \tag{5.27}$$

And secondly, we may require that $\lambda(\varphi) \equiv \lambda_\varphi$ must be consistent with both

$$\upsilon(\varphi) = \upsilon_\varphi = \upsilon_0/\Gamma_e = \text{constant}, \tag{5.28}$$

and the intrinsic propagation speed of the photon,

$$c w^0_\varphi = \upsilon_\varphi\, \lambda_\varphi. \tag{5.29}$$

Thirdly, from the EM gravity expression for $p^i$, the mass-energy of an excited atom which is at rest in the field is Γ times its rest value, $E^0$. Hence, the photon is emitted with an energy

$$E_e = \Gamma_e E^0 = \Gamma_e\, hc/\lambda_0 = hc/\lambda_e. \tag{5.30}$$

From these definitions we then obtain

$$\lambda_e = \lambda_0/\Gamma_e, \tag{5.31}$$

$$w^0_\varphi = g_{00} = 1/\Gamma^2, \tag{5.32}$$

$$\lambda_\varphi = \lambda_0\Gamma_e/\Gamma^2, \tag{5.33}$$

and finally,

$$E_\varphi = (h\upsilon_0)\Gamma^2/\Gamma_e. \tag{5.34}$$

In accordance with this scheme, the amount of energy that a photon looses in overcoming a potential $\Gamma_e$, with a constant $\Sigma$–frequency, is then given by

$$\Delta E = (h\upsilon_0)(\Gamma_e - 1/\Gamma_e), \tag{5.35a}$$

$$= 2\varphi_m\Gamma_e(h\upsilon_0) = 2\varphi_m E_e, \tag{5.35b}$$

where $\varphi_m = \frac{1}{2}\mathbf{u}^2 = -\Phi_n/c^2$.

F. The EM field-equations describing how the $g_{ik}(u^\alpha)$ potentials are sourced by the (1 + 3) energy-momentum tensor of mass-energy (exercises for the student).

1. The procedure

Using the $\Gamma^i_{jk}(u^\alpha)$ from Appendix A, evaluate the general form of the field equations of gravitation

$$f(u^\alpha,\varphi_m)_{jk} = \frac{\partial}{\partial x^i}\Gamma^i_{jk} + \Gamma^i_{jm}\Gamma^m_{ki} = -\kappa(T_{jk} - \tfrac{1}{2}g_{jk}T), \tag{5.36}$$

as given by Einstein[32] for the special (greatly simplified) case that $g = -1$, which is true for $g_{jk}(u^\alpha)$, to get EM field equations in terms of second order time and 3-space derivatives of $u^\alpha$ and $\varphi_m$.

2. Challenges for mathematicians.

Solve the EM field equations to get $u^\alpha(x^i)$ for the case that $T_{ik}$ represents a spherically symmetric body at rest, so that the field is that of ideal Newtonian gravity. And use the equations, $\beta = \mathbf{u} + \mathbf{w}$, and $\beta^0 = \mathbf{u} + \mathbf{w}^0$, with and without $w^0 = (1 - \mathbf{u}^2)$, to calculate the angular precession of a planet's perihelion and the gravitational deflection of light. Optionally, use (5.42).

3. Some completeness Issues for the student of EM gravity.

Can you devise a more general definition of ds that puts $\nabla\times\mathbf{u}$ and/or $\nabla\bullet\mathbf{u}$ terms into $g_{jk}$?

Can you devise a thermodynamic derivation of the way that lightspeed should vary in the field, $w^0_\varphi$? And if so, can you incorporate that into a more general definition of ds that makes explicit use of $w^0_\varphi$, say by normalizing w to $w^0_\varphi$ rather than c?

For experts in tensor analysis, Taub has constructed a very elegant Relativistic Fluid Mechanics[33] for the $\gamma$-coupling domain where $g_{jk}$ (his $\eta_{\mu\nu}$) = $-G_{ik}$ = diag.(−1, 1, 1, 1). Consider revising his notation so that the signature of the fundamental tensor is diag.(1, −1, −1, −1), and then let the medium be the $\in_m$–continuum with $\eta_{\mu\nu} = g_{jk}(u^\alpha)$. Taub notes that in the limit where a mass-energy medium becomes incompressible, the speed of sound $\rightarrow$ the speed of light. Does that mean one could treat the $\in_m$–continuum as a medium that is effectively incompressible in the limit $u \rightarrow 0$?

F. Some fruitful reinterpretations of quantum theory and the experimental data of high-energy particle physics.

To be completed.

## VI. GFT3: The General Physical Principles and Field Equations of Hyper-Energy Theory.

### A. Introduction

The 20th century disciplines of particle physics and cosmology uncovered six *profound structural and dynamical attributes* (PSADAs(1−6)) of Einstein's empty ( ${}^{0}\in_{m} = 0$) 3−space. However, since Maxwell's energy rich $\in_{m}$−continuum (${}^{0}\in_{m} >> 0$) supersedes empty 3−space, it follows that the PSADAs(1−6) of empty 3−space will be both more correctly and more profitably interpreted as six *profound structural and dynamical attributes* of Maxwell's ${}^{0}\in_{m}$−continuum.

It will be shown, for example, that given GFT1, PSADAs(1−2) of the ${}^{0}\in_{m}$−continuum are entirely sufficient to identify both the general (1 + p)-dimensional principles of *hyper energy theory* and the general type of field equations of *hyper-energy theory;* with *a priori* assurance that the theory is fully consistent with GFT1, GFT2, and the revolutionary 20th century findings of particle physics and cosmology. PSADAs(2−6) of the ${}^{0}\in_{m}$−continuum will then be seen to identify five structural and dynamical attributes of the ${}^{0}\in_{m}$−continuum that *hyper-energy theory* must account for. More precisely:

- PSADAs(1−2) will be shown to identify the general (1 + p)-dimensional physical principles of hyper-energy theory and the general (1 + p)-dimensional form ($\psi_{g}$) of its field equations; in terms of five *hyper-energy postulates* (HEPs(1−5)); thereby causing the (1 + 3)-dimensional properties of the ${}^{0}\in_{m}$−continuum (that were implied by GFT1) to be viewed as compactification solution of the $\psi_{g}$.
- PSADAs(2−6) will then be seen as five specific dynamic and structural attributes of the ${}^{0}\in_{m}$−continuum that hyper-energy theory must explain.

Sections VII − IX are then devoted to illustrating—with the help of some crucial *p-invariant solution characteristics*—hyper-energy theory's innate potential to explain PSADAs(2−6), and thus, by implication, everything else about this classically compactified and quantized universe.

### B. PSADAs(1−2) and HEPs(1−5).

#### 1. PSADA1 (Uncovered via the classically-grounded quantum theory of superstrings.)

The revelation of superstring theory that physical reality has a (1 + p)–dimensional parameterization with p = 9, has evolved over the last 90 years from the ground breaking publications by Gunnar Nordström (in 1914)[34] and Theodore Kaluza (in 1921)[35] which suggested that *classical electromagnetism and gravity* (CEMG) could be described as different aspects of a (1 + 4)-dimensional CEMG field. The history of the evolution of p = 4 to p = 25 and back down to p = 9 (driven by the desire to have the *weak* and the *strong* nucleon forces included in unified formulation), and the persistent absence of a suitable (1 + p)→'(1 + 3)'-dimensional *compactification theory*, are well documented.[36, 37, 38, 39, 40, 41]

Because superstring theory unifies a wealth of hard-earned data regarding $\delta\mathcal{E}_{m}$ particle transmutations and interactions, there seems to be little doubt that a (1 + p > 3)-dimensional reality and a significant fraction of superstring theory "*will survive in the final underlying laws*".[42]

We can therefore claim to know the following about the ${}^{0}\in_{m}$−continuum:

PSADA1: The ${}^{0}\in_{m}$−continuum of this universe is a (1 + p > 3)-dimensional energy structure that imposes a (1 + 3)-dimensional constraint on the dynamics and interactions of its $\delta\mathcal{E}_{m}$ particles via a *compactification physics* that is indigenous to hyper-energy theory.

Given GFT1 and PSADA1 we are naturally compelled to set down the following first postulate of hyper-energy theory:

HEP1: Maxwell's ${}^{0}\in_{m}$–continuum is a (1 + p > 3)-dimensional energy structure that is fabricated out of an elementary, (1 + p)-dimensional, non particulate, inviscid, and compressible *energy substance,* called *hyper-energy,* in such a way as to impose a (1 + 3)-dimensional constraint on the dynamics and interactions of its (1 + p)-dimensional $\delta\mathcal{E}_m$ particles.

2. PSADA2 (Uncovered via the cold, inflationary, expansion phase of big bang cosmology.)

In accordance with contemporary *big bang cosmology* (BBC) and HEP1, we can now claim to also know the following about the ${}^{0}\in_{m}$–continuum:

PSADA2: The ${}^{0}\in_{m}$–continuum of this universe came into being about 14 billion years ago, *entirely devoid of any* $\delta\mathcal{E}_m$ *particles* and initially expanding at a relatively high (so-called *inflationary*) rate, seemingly as a direct result of a single point-like explosion in the (1 + p)-dimensional continuum of hyper-energy.

Although HEP1 qualitatively answers the presently open BBC question of what existed before the big bang, it does not address the more fundamental open question of what caused the big bang. We lay down the following second postulate of hyper-energy theory in order to provide plausible answers to both of these questions:

HEP2: Prior to the big bang there was, and still is, a permanent (1 + p)-dimensional continuum of hyper-energy which is here postulated to possess a presently unknown form of internal energy (very likely vibrational) which has a natural tendency to produce stochastic eruptions that are referred to, for lack of a better terminology, as *percolations* or *percs* representing point-like explosions in the hyper-energy continuum.

We therefore propose that a hyper-energy *perc* provides a qualitative energy-conserving explanation for *a big bang*.[a] Accordingly we are led to lay down the third hyper-energy postulate as a partial answer to the question implied by HEP1; regarding the precise way in which the ${}^{0}\in_{m}$–continuum *was fabricated* from the hyper-energy continuum. Namely:

HEP3: The ${}^{0}\in_{m}$–continuum originated from one or more *percs* in the hyper-energy continuum.[a]

In Section VII the expansion dynamics of the ${}^{0}\in_{m}$–continuum is modeled in a way that depends on only *the fact of a perc* and not on *the detailed physical reason(s) for it*. Nevertheless, it might be heuristically useful to consider the following speculations on the physics underlying hyper-energy percs:

*a. Non-critical heuristic speculations on perc physics.*

It is reasonable to assume that the vibrational internal energy of hyper-energy sources the propagation of various types of energy waves which effectively endow the hyper-energy continuum with a potential, $\Phi[x^a(1 + p)]$,[b] for stochastically generating hyper-energy percs at points of convergent, constructive interference.

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[a] In Section VII, we will be explaining how (p – 3) percs could acount for a $(1 + p)\rightarrow$'(1 + 3)'-dimensional compactification.
[b] The index (a) covers the integer range (0–p).

3. The general field equations of hyper-energy theory.

We are thus led to lay down the following fourth postulate of hyper-energy theory:

HEP4: The general field equations of hyper-energy theory ($\psi_g$) are, to a first order of approximation, a (p − 3)-dimensional extension of the ordinary (1 + 3)-dimensional, Lorentz covariant, nonlinear mathematics of *compressible fluid dynamics;* applied to the abstraction of a (1 + p)-dimensional continuum of inviscid, non particulate, hyper-energy; [43, 44, 45, 46] and encompassing the related but much newer disciplines of Soliton Theory and Chaos Theory.[47, 48, 49, 50]

4. This universe as a hierarchy of solutions ($S_k$) to specific k-sectors ($\psi_k$) of the $\psi_g$.

In concert with most field theories, we say that a *k–manifestation sector* of hyper-energy is defined by a *k-set* of *delimiting physical assumptions* (boundary conditions, sources, symmetries, parameter constraints, etc.) which transform $\psi_g$ into a particular subset or *k-sector*, $\psi_k$, the various $i^{th}$ solutions of which, $S_{ki}$, yield various *k–type manifestations of hyper-energy*. The k-index is generally alpha-numeric.

In view of HEPs(1−4) then, we are led to lay down the following fifth postulate of hyper-energy theory:

HEP5: The creation of the ${}^0\in_m$–continuum of this universe, its subsequent expansion dynamics, its internal $\delta\mathcal{E}_m$ field-particle evolution, and the interaction properties of all of its $\delta\mathcal{E}_m$ field-particles, can be mathematically described via a hierarchy of solutions to specific sectors of the $\psi_g$ that are driven, at the lowest level of complexity and the highest level of potential, by one or more hyper-energy percs.

C. PSADAs(3−6) (As four additional specific challenges to the $S_k$ of hyper-energy theory.)

1. PSADA3 (Uncovered via the Hubble expansion phase of BBC.)

Given the present BBC, PSADAs(1−2), and HEPs(1−5), we can now claim to also know the following about the ${}^0\in_m$–continuum:

PSADA3: The initial *inflationary* expansion rate of the ${}^0\in_m$–continuum spontaneously transitioned to a much slower and constant *Hubble rate* with a change in its internal structure that caused a fixed number of primordial Hydrogen, Helium, and Lithium atoms, and photons, to be precipitated.[a]

2. PSADA4 (Uncovered via theoretical investigations into the physical nature of time.)

In the spring of 1963, with support from the Air Force Office of Scientific Research, 22 leading physicists met at Cornel University for two days to brainstorm the question: W*hat is the physical nature of time?* Commenting on the outcome of the colloquium, Thomas Gold, the moderator of the colloquium, stated:

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[a] Manifesting a $\delta\mathcal{E}_m$ *particle temperature* of ≈ 4,000 K at a time of ≈ 100,000 years after the big bang**,** when the photons of the present 2.7 K *cosmic microwave background* (CMB) were first able to propagate relatively freely throughout the expanding ${}^0\in_m$–continuum. How this fixed supply of elemental atoms and photons became the complex universe of matter-energy and life-forms that exists today is a truly amazing story that has been summarized by many authors (see refs 36–39 and references sited therein).

"*We can not claim to have solved the main problems…and it may be that no very profound improvement in our understanding of time will ever take place; but it could also be that some new physical theory will be devised one day that depends on, or defines, a different concept of time…I personally think that our present understanding is inadequate and is perhaps holding us back from a better description of nature.*"[51]

This claim notwithstanding, the group did come up with a general qualification of time, and a corollary of it, which said *new physical theory* might be able to fully explain. As summarized by Bondi:

*"If it is agreed that the expansion of the universe has a lot to do with the problem of time, a conclusion which I regard to virtually inescapable because this process leads to the dark night sky, the disequilibrium between matter and radiation, and to the fact that radiated energy is effectively lost, then we accept a very close connection between cosmology and the basic structure of our physics."*[52]

Hence, we can now claim to also know that:

PSADA4: The physical nature of time, and the structures and interactions of all $\delta\mathcal{E}_m$ particle, are all intimately related to the expansion of the ${}^0\in_m$–continuum of this universe.

3. PSADA5 (Uncovered via analyses of the elementary particle spectrum.)

The elementary particle spectrum encompasses both the long-life stable particles (and anti-particles) and the hundreds of mostly short-lived particles (and anti-particles) that 20$^{th}$ century science discovered can be created from the *kinetic mass-energy* unleashed in head-on collisions of *parent particles.* The end products of most high energy collision experiments are thus more *ordinary* (stable) $\delta\mathcal{E}_m$ *particles* than were present before the experiment.

From the seasoned perspective of Werner Heisenberg[53] this implies that a *fundamental theoretical understanding* of the particle spectrum must be possible, and along the same general, nonlinear, field-theoretic lines that so many other spectra have come to be understood. According to Heisenberg this implies that theorists should be earnestly seeking to formulate the following two things:

- A structure of the *so-called 'empty space'* that provides an intrinsic *boundary condition* for the types of particles that can be transiently or permanently created from kinetic mass-energy.
- An underlying dynamics of matter.

These challenges, Heisenberg said, should be addressed *without any philosophical prejudices,* and the resulting theory *must be taken seriously*—provided that it rests on *well defined hypotheses that leave no essential points open.*

"*The particle spectrum can be understood only if the underlying dynamics of matter is known; dynamics is the central problem.*"[54]

We have already deduced that a formulation of the fluid dynamics of hyper-energy underlying the ${}^0\in_m$–continuum and $\delta\mathcal{E}_m$ particles *is central.* But now we can claim to also know the following about the ${}^0\in_m$–continuum:

PSADA5: The ${}^0\in_m$–continuum contains an intrinsic boundary condition that serves to delineate the various types of $\delta\mathcal{E}_m$ particles which can be formed from collisionally unleashed kinetic mass-energy.

4. PSADA6 (Uncovered via additional analyses of the elementary particle spectrum.)

From the seasoned perspective of elementary particle theorist, Hans Peter Dürr:

*"It seems that here we come face to face with very general and most important relations that we had failed to take into consideration.*

*"If one of nature's fundamental symmetries is regularly found to be disturbed in the spectrum of elementary particles, the only possible explanation is that the universe, i.e., the substratum where the particles originated, is less symmetrical than the underlying physical law. It follows that there must be forces acting over long distances, or elementary particles of vanishing inertial mass.*

*"This is probably the best way of interpreting electrodynamics. Gravitation, too, could arise in this way, so that here we may hope to find a bridge to the principles on which Einstein wanted to base his unified field theory and cosmology…*

*"At present it looks very much as if we can interpret the whole of electrodynamics in terms of the asymmetry of the universe vis-à-vis the proton-neutron exchange or more generally vis-à-vis the isospin group."* [55]

Hence, we can now claim to also know the following about the ${}^{0}\!\in_{m}$–continuum:

PSADA6: The ${}^{0}\!\in_{m}$–continuum contains a substratum which is characterized by a basic physical asymmetry in its (p − 3)-dimensions, in which both electromagnetism and gravity appear as *long range* pseudo (1 + 3)-dimensional *fields.*

D. Additional challenges for the $S_k$ of hyper-energy theory.

In addition to PSADAs(2–6) of the ${}^{0}\!\in_{m}$–continuum, we have $E = mc^2$, $\underline{\underline{w}}d^{1-5}$, and the quantum physics of $\delta\mathcal{E}_m$ particles as additional challenges for the $S_k$ of hyper-energy theory.

E. Valuable insights into the formal $S_k$ of hyper-energy theory stemming from p-invariant symmetries and analogous sectors of ordinary hydrodynamics.

In Sections VII – IX, we exploit some symmetry-rooted, p-invariant, qualitative features of the $S_k$, and their well studied (1 + 3)-dimensional isomorphs, $S'_k$, to illuminate hyper-energy theory's innate ability for addressing all of the above described modeling challenges, via the following hierarchy of *hyper-energy solution sectors:* [a] [b]

- The *spherical hypershock* ($\psi_{sh}$, $S_{sh}$) sector of the $\psi_g$.
- The long range $\delta\mathcal{E}_m$-*particle structure* ($\psi_{ps}$, $S_{ps}$) sub-sector of $S_{sh}$.
- The massive and massless *soliton*-$\delta\mathcal{E}_m$-*particle propagation* ($\psi_{spp}$, $S_{spp}$) sub-sector of $S_{sh}$-$S_{ps}$.
- The massless *dispersion*-$\delta\mathcal{E}_m$-*wave propagation* ($\psi_{dwp}$, $S_{dwp}$) sub-sector of $S_{sh}$-$S_{ps}$.
- The *de-Broglie wave* ($\psi_{dBw}$, $S_{dBw}$) sub-sector of $S_{sh}$-$S_{ps}$.
- The *quantum behavior* ($\psi_{qb}$, $S_{qb}$) sub-sector of $S_{sh}$-$S_{ps}$-$S_{dBw}$.

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[a] Thereby providing mathematicians with six reasonably well defined converging paths to the final field equations of hyper-energy theory and a qualitative understanding of some of the revolutionary technologies that may be then brought into being.

[b] As illustrated, we prime (′) all aspects of mass-energy fluid dynamics which are similar to hyper-energy fluid dynamics.

## APPENDIX A.

## THE CHRISTOFFEL FUNCTIONS (OF THE SECOND KIND) OF $g_{ik}(u^\alpha)$

*******Derivations to be added before publication.**************

1. The Christoffel functions (of the second kind) of $g_{ik}(u^\alpha)$ are defined by:

$$\Gamma^i_{jk} = \Gamma^i_{kj} = \Gamma^i_{jk}(\partial g_{jk}/\partial x^i) = \tfrac{1}{2} g^{im}[\partial g_{jm}/\partial x^k + \partial g_{mk}/\partial x^j - \partial g_{jk}/\partial x^m]. \qquad \text{(A−1)}$$

With $g_{jk}(u^\alpha)$ and $g^{jk}(u^\alpha)$ given by (4.13), and $\varphi = \varphi_m = \tfrac{1}{2}\mathbf{u}^2$, and $\partial_k \equiv \partial/\partial x^k$, I obtained:

$$\Gamma^0_{00} = \mathbf{u}\bullet\nabla\varphi, \qquad \text{(A−2)}$$

$$\Gamma^0_{0\alpha} = [-\nabla\varphi + \tfrac{1}{2}\mathbf{u}\times(\nabla\times\mathbf{u})]^\alpha, \qquad \text{(A−3)}$$

$$\Gamma^0_{\alpha\beta} \equiv D_{\alpha\beta} = \tfrac{1}{2}(\partial_\alpha u^\beta + \partial_\beta u^\alpha), \qquad \text{(A−4)}$$

$$\Gamma^\gamma_{00} = -[\nabla\varphi + \partial_0\mathbf{u} - \mathbf{u}(\mathbf{u}\bullet\nabla\varphi)]^\gamma, \qquad \text{(A−5)}$$

$$\Gamma^\gamma_{0\beta} = -\tfrac{1}{2}u^\gamma[\nabla\varphi + (\mathbf{u}\bullet\nabla)\mathbf{u}]_\beta + \{\tfrac{1}{2}[\partial_\gamma u^\beta - \partial_\beta u^\gamma] \equiv C^\gamma{}_\beta\}, \qquad \text{(A−6)}$$

$$\Gamma^\gamma_{\alpha\beta} = u^\gamma\{\tfrac{1}{2}[\partial_\alpha u^\beta + \partial_\beta u^\alpha] \equiv D_{\alpha\beta}\} = u^\gamma D_{\alpha\beta}, \qquad \text{(A−7)}$$

Hence, $\Gamma^k_{\alpha\beta} = (1, u^\gamma)D_{\alpha\beta} \equiv u^k D_{\alpha\beta}$. (A−8)

### 2. Subsidiary Formulas

In the process of deriving and implementing these components of $\Gamma^i_{jk}$, use was made of the following formulas:

$$(\mathbf{u}\bullet\nabla)\mathbf{u} = \nabla\varphi - \mathbf{u}\times(\nabla\times\mathbf{u}). \qquad \text{(A−9)}$$

For any 3-vector, $\mathbf{A}$, $A^\alpha(2\,C^\gamma{}_\alpha) = [(\mathbf{A}\times(\nabla\times\mathbf{u})]^\gamma$. (A−10)

For any three 3-vectors $(\mathbf{a},\mathbf{b},\mathbf{c})$; $\mathbf{b}\bullet[(\mathbf{a}\bullet\nabla)\mathbf{c}] = \mathbf{a}\bullet[(\mathbf{b}\bullet\nabla)\mathbf{c}] + \mathbf{a}\bullet[\mathbf{b}\times(\nabla\times\mathbf{c})]$. (A−11)

Therefore, $\mathbf{a}\bullet[(\mathbf{u}\bullet\nabla)\mathbf{u}] + \mathbf{a}\bullet[\mathbf{u}\times(\nabla\times\mathbf{u})] = \mathbf{a}\bullet\nabla\varphi$, (A−12)

and $\mathbf{u}\bullet[(\mathbf{a}\bullet\nabla)\mathbf{u}] = \mathbf{a}\bullet\nabla\varphi$. (A−13)

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[29] This raises an important general question concerning tensor analysis in general and (1 + 3) tensor analysis, in particular, which I will simply state and not attempt to explore in any great detail, here. Namely, what are the physical significances of various *covariant* (1 + 3) quantities? Or, more generally—regarding any physics that is being described—how much extra information is made available by dividing the physics into its *contravariant* and *covariant* forms? Because it has a lot to do with the physics of ${}^0\in_m$ and its 100% coupling of mass-energy structures to ${}^0\in_m$, I would venture to say, a great deal.

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