IV. Einstein-Maxwell Gravity.

A. Einstein's foresight

It is well known that Einstein never wavered from his conviction that the unification of cosmology and particle physics would come about via a continuum theory that would be able to account for cosmology and the structures of *all* particles. Einstein defended this proposition with enthusiasm in his correspondences—referring to his own efforts to bring this about, interestingly enough, as *his* "Maxwellian Program". (Holton, 1996)

In (Holton, 1996) we "*hear"* Einstein arguing that *we can not reasonably settle for a wave-particle duality in nature* that gives more or less *equal status* to both *the field* and to its antithesis*, discrete matter-energy*. And for Einstein, there was no question as to which of these two extremes had the greatest logical probability of being both necessary and sufficient to explain everything. As he said in a letter to his old friend, Michele Besso,

> "I consider it quite possible that physics might not, finally, be founded on the concept of field—that is to say, on continuous elements. But then out of my whole castle in the air—including the theory of gravitation, but also most of current physics—there would remain *nothing.*" (Holton, 1996)[24]

Einstein also believed that any such classical field theory and cosmology would necessarily incorporate his general theory of relativity in some manner.

B. The mass-energy equivalence principle and tensor mathematics furnished by Einstein.

1. Einstein's $E = mc^2$ unification of particle energy and mass.

Einstein's unification in 1905 of the two previously disparate physical concepts of particle energy and particle mass greatly enlarged the then fairly new and restricted 19th century conception of energy; endowing energy with a *universality* and *objective substance* that was previously unknown.

The entire domain of matter particles and fields began to be viewed thereafter in an entirely new fundamental way: As different manifestations of a universal type of *energy,* called *mass-energy*, which exhibits the *mechanical property* of inertia that, previously, only *bodies of matter* possessed. Thus making it possible to conclude that, in retrospect:

> *Energy* is a more fundamental *classical substance* for the discipline of *fluid mechanics* than the passive and inert property of *mass* on which fluid mechanics—the very model and inspiration for *field theory,* to begin with—was originally founded.

The fact that all matter particles and fields consist of *mass-energy*, is in qualitative agreement with implication (i) of Maxwell's SE Gravity theorem; that the $\delta\mathcal{E}_m$ energy of any matter particle or field represents a miniscule perturbation of ${}^0\mathcal{E}_m$, denoted symbolically by $\delta\mathcal{E}_m$.[25] One can thus assume that the $(1 + p)$-dimensional *hyper-energy* explanations for ${}^0\mathcal{E}_m$, its $(\delta\mathcal{E}_m)$s, and thus, $\delta\mathcal{E}_m$ propagation, will explain both the $(1 + 3)$ *mass* (m) and associated mc^2 *energy* of any $\delta\mathcal{E}_m$ in full agreement with the implication (iv); that c^2 is an indirect measure of ${}^0\in_m$. These assumptions are qualitatively vindicated with the compactification model described in Sections VII – IX.

2. The (1 + 3) tensor calculus of covariant and generally covariant physical laws.

At the turn of the 20^{th} century, the labors of a number of mathematicians throughout the last half of the 19^{th} century had produced the elements of a *static* p–dimensional tensor calculus.[26] And Minkowski deserves credit for pointing out, in 1908, that special relativity theory could be elegantly presented and utilized in the context of a 4-dimensional *geodynamic* (3 + 1) subset of the tensor calculus.[27]

Einstein's special theory of relativity can thus be rightfully credited with causing a unique and powerful (1 + 3) branch of the (0 + *n*)–dimensional tensor calculus to become established as an inviolably economical and practical tool for developing descriptions of *mass-energy physics*—in all of its many diverse domains of expression; from high energy particle physics to the nonlinear fluid thermodynamics of numerous practical *mass-energy* continua—descriptions which are guaranteed to be intrinsically Lorentz invariant, and thus, as correct as they can possibly be—at the (1 + 3) level of approximation of physical reality. And of course Einstein himself found the (1 + 3) tensor calculus useful for addressing the problem of gravity in a manner consistent with his perceived ($^0\in_m = 0$) view of 3-space.

Hence, the only thing that could make it better would be if general relativity theory—which is known to be actually *too general*—could be mathematically restricted in a way that would make the resulting descriptions completely compatible with Maxwell's gravity theorem. If this could be done it would create a clear (1 + 3)-dimensional Einstein-Maxwell interface to a (1 + *p*)-dimensional hyper energy physics of the future that would bring closure to the goal that Einstein and Maxwell both shared; for a basically *classical* unified field theory. A goal which Einstein and Maxwell laid complementary mathematical foundations for via their respective mathematical theories of electromagnetism and gravity. The remainder of this section will show that this grand EM unification is not only possible but quite inevitable.

C. The *implicit* 4-vector potential of gravity defined by general relativity theory.[28]

The general relativistic analysis of gravity is founded on the (1 + 3) tensor formula

$$ds^2 = g_{ik}dx^i dx^k = dx^i dx_k, \tag{4.1}$$

where the $g_{ik}(x^i)$ are presumed to contain 10 unknown *gravitational potentials*—sourced by the 3–volume density of mass-energy and mass-energy-momentum (T_{ik}), in accordance Einstein's equations,

$$R_{ik} - ½Rgik = CT_{ik}. \tag{4.2}$$

This wholly *metric* description of gravity happens to incorporate an *inherently dynamical* 4–vector potential which surfaces when Eq. (4.1) is put into the following equivalent *dynamical* form:

$$ds^2 = g_{ik}dx^i dx^k = \beta^i\beta_i\,(dx^0)^2, \tag{4.3}$$

where,

$$\beta^i = dx^i/dx^0 = (1, \beta), \text{ and } \beta_i = dx_i/dx^0 = (\beta_0, \beta_\alpha) = (\beta_0, \vec{\beta}).^a \tag{4.4}$$

From (4.3) and (4.4), it can be seen that *all* of the field-theoretic information about the field of gravity—for any conceivable physical situation (irrespective of how the field is sourced)—is stored in nothing more than, and nothing less than, the four elements of

[a] As indicated, we will be using an *over arrow* to denote the *3-vector* part of a *covariant* 4–vector.

$$\beta_i = (\beta_0, \beta_\alpha) = g_{ik}\beta^k.^{29} \tag{4.5}$$

The particular way that the gravity potentials are stored in β_i can be seen most clearly by defining $g^\alpha = g_\alpha = g_{0\alpha}$, and $G^\alpha = G_\alpha = g_{\alpha\gamma}\beta^\gamma$, so that the components of β_i can be expressed as

$$\beta_0 = (g_{00} + \mathbf{g}\bullet\beta), \tag{4.6}$$

$$\vec{\beta} = (\mathbf{g} + \mathbf{G}). \tag{4.7}$$

Hence, all of the (1 + 3) information about the gravitational potentials—for *any conceivable gravitational field*—is contained in this particularly simple looking mathematical structure. It thus seems clear that the completion of Einstein's Maxwellian program and a solution to quantum gravity problem can now be precipitated by simply

- Deriving an explicit expression for β_i which is uniquely consistent with Maxwell's gravity theorem.
- Allowing the resulting *dynamical representation* of ds^2 (the right side of (4.3)) to be the *fundamental scalar* that the *geometrical representation* (the left side of (4.3)) is slaved to.

This is precisely what is done next.

D. The dynamical scalar illuminated by flat spacetime.

In the γ–coupling domain of the $\in_m$-continuum, Eq. (4.3) becomes

$$ds^2 = G_{ik}dx^i dx^k = (\beta_i\beta^i)(dx^0)^2 = (1 - \beta^2)(dx^0)^2 \equiv (dx^0/\gamma)^2 = [(dx^0)^2 - d\mathbf{r}^2], \tag{4.8}$$

where $\beta_i = (1, -\beta)$, $g_{ik} \rightarrow G_{ik} \equiv \text{diag.}(1,-1,-1,-1)$, and $\gamma \equiv 1/[1 - \beta^2]^{1/2}$.

From the practical perspective of Σ, ds is a valid 4-scalar because it attains an absolute countable value of zero for the special case that d**r** refers to a point of electromagnetic radiation—as required by Maxwell's theory of electrodynamics. And the successes of Einstein's special relativity theory have assured us that the 4-scalar status of ds is preserved, as one might logically expect, when d**r** refers to a p^π particle as well.

1. The expanded (1 + 3) dynamical symmetry underlying Einstein-Maxwell gravity.

The question then is, how can the *dynamical symmetry* of $\beta^i\beta_i$ be expanded so that it converges to $(1 - \beta^2)$ for $\mathbf{u} = \mathbf{0}$, and is otherwise in agreement with **w-**gauge theory? The answer seems to be, simply substitute ℓ for γ in (4.8) so that $\beta^i\beta_i$ has the explicit form

$$\beta^i\beta_i(p) = (1 - \mathbf{w}^2) \equiv 1/\ell^2, \tag{4.9a}$$

$$= 0 \text{ if } \mathbf{w}(p) \rightarrow \mathbf{w}(p^0) \equiv \mathbf{w}^0, \tag{4.9b}$$

$$\geq 0 \text{ if } \mathbf{w}(p) \rightarrow \mathbf{w}(p^\pi) \equiv \mathbf{w}. \tag{4.9c}$$

The *propagation velocity* of $\delta\mathcal{E}_m$ field-particles in the $\in_m$-continuum then takes over the symmetry role that was previously played by the *ordinary* rectilinear velocity β of matter particles in *empty 3–space*. Now that β, in accordance with Eq. (2.3), is physically explained by *either one* or *both* of two fundamental transport mechanisms (as $\beta = \mathbf{w} + \mathbf{u}$), β^2 is no longer fixed at unity for a p^0 field-particle. Hence something else is needed to create an absolute countable null value for ds when electromagnetic radiation is being described. And **w**—with its null 3-scalar characterization of ${}^{0}\in_m$ (per Section II.D.3)—seems to be perfectly suited for doing just that.

From the viewpoint of Σ^d then, $\mathbf{w}(p) = \beta(p) - \mathbf{u}(x^i)$ describes the 3-vector difference between the coordinate velocity of $p(x^i)$ and the flow velocity of $\in_m$ at (x^i). Thus, wherever it is absolutely necessary or heuristically useful to do so, we will expand **w** into $\beta - \mathbf{u}$, $\underline{\mathbf{w}}$ into $\underline{\beta} - \underline{\mathbf{u}}$, and $\underline{\underline{\mathbf{w}}}$ into $\underline{\underline{\beta}} - \underline{\underline{\mathbf{u}}}$. Otherwise we will exploit the compactness of w, $\underline{\mathbf{w}}$, and $\underline{\underline{\mathbf{w}}}$, and their various inner products in the equations which evolve as we proceed to explore the consequences of employing Eq. (4.9a) as a driving dynamical symmetry for EM gravity.

2. The *explicit* 4-vector potential of Einstein-Maxwell gravity.

To determine the $g_{ik}(\Phi^i)$ consistent with Eq. (4.9a) we set

$$\beta^i\beta_i(p) = [1 - (\beta - \mathbf{u})^2] = [(1 - \mathbf{u}^2) + 2\mathbf{u}\bullet\beta - \beta^2], \tag{4.10}$$

and compare this with the right sides of Eqs. (4.6-7) to obtain the unique Einstein-Maxwell components of β_i as,

$$\beta_0 = [(1 - \mathbf{u}^2) + \mathbf{u}\bullet\beta)] = (g_{00} + \mathbf{g}\bullet\beta). \tag{4.11a}$$

$$\vec{\beta} = (\mathbf{u} - \beta) = -\mathbf{w} = (\mathbf{g} + \mathbf{G}). \tag{4.11b}$$

Hence,

$$g_{oo} = (1 - \mathbf{u}^2) \equiv (1 - 2\Phi_m) = (1 + 2\Phi_n/c^2), \tag{4.12a}$$

$$\mathbf{g} = \mathbf{u}, \tag{4.12b}$$

$$\mathbf{G} = -\beta, \tag{4.12c}$$

$$g_{\alpha\gamma} = -\delta_{\alpha\gamma}. \tag{4.12d}$$

In matrix form:

$$g_{ik} = \begin{array}{c} i\ j\rightarrow \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{cccc} 0 & 1 & 2 & 3 \\ \end{array} \begin{bmatrix} 1 - \mathbf{u}^2 & u^1 & u^2 & u^3 \\ u^1 & -1 & 0 & 0 \\ u^2 & 0 & -1 & 0 \\ u^3 & 0 & 0 & -1 \end{bmatrix}, \tag{4.13a}$$

$$g^{ik} = \begin{array}{c} i\ j\rightarrow \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{cccc} 0 & 1 & 2 & 3 \\ \end{array} \begin{bmatrix} 1 & u^1 & u^2 & u^3 \\ u^1 & (u^1)^2 - 1 & u^1u^2 & u^1u^3 \\ u^2 & u^2u^1 & (u^2)^2 - 1 & u^2u^3 \\ u^3 & u^3u^1 & u^3u^2 & (u^3)^2 - 1 \end{bmatrix}, \tag{4.13b}$$

where, $g = |g_{ik}| = |g^{ik}| = -1$.

Hence, given $g_{ik}(u^\alpha)$ and the EM 4-GRV

$$\nu^i = dx^i/ds = (\nu^0, \nu) = \ell\beta^i, \tag{4.14}$$

the many consequences of EM gravity can then be derived from the perspective of Σ^d, via the equation

$$ds^2 = (1 - \mathbf{w}^2)(dx^0)^2 = g_{ik}(u^\alpha)dx^i dx^k = dx^i dx_k = \beta^i\beta_i\,(dx^0)^2 = \nu^i\nu_i\,ds^2 \equiv (dx^0/\ell)^2. \tag{4.15}$$

Operationally it is clear that this equation merely dictates the form of an invariant *4-rate of change operator*, $d/ds = \ell d/dx^0$, which, in turn, dictates the contravariant and covariant forms of the equations describing the first and second order, nonlinear thermodynamics of a free p^{π} particle in an EM gravity field parameterized by $g_{ik}(u^{\alpha})$, irrespective of the actual distribution of $\mathbf{u}(x^i)$—*analogous to the source free equations of Maxwell's electrodynamics*.

In the view of Maxwell's equations and the lightspeed scalar, S_3, it is also clear that the invariance of $[(dx^0)^2 - (d\mathbf{r})^2]$ for free electromagnetic energy and p^{π} particles expressed by

$$ds^0 = d\underline{s}^0 = 0, \tag{4.16a}$$

$$ds = d\underline{s} \geqslant 0, \tag{4.16b}$$

serve as a quantification of NPA–C2.

E. The general (1 + 3) EM **w**-deformation of scalar time measures.[a]

Let $\underline{\underline{v}}^i = \underline{\underline{\ell}}\underline{\underline{\beta}}^i$ be the 4–GRV of both $\underline{O}$ and $\underline{\mathbf{r}}$. The reference point $`\underline{\mathbf{r}}$ and a $\delta\mathcal{E}_m$ particle at x^i of Σ^d are then embedded in the same $\in_m$ flow field, so that formally we can set $\underline{\mathbf{u}} = \mathbf{u}$, $\underline{\underline{\mathbf{w}}} = \underline{\underline{\beta}} - \mathbf{u}$, and $\mathbf{w} = \beta - \mathbf{u}$. The empirical time change $d\underline{x}^0$ can then be unambiguously expressed in terms of Σ parameters via the *velocity-coupling pseudo scalar* ($\underline{\underline{v}}_i\beta^i = \underline{\underline{v}}^i\beta_i$) via the equation

$$d\underline{x}^0 = (\underline{\underline{v}}_i\beta^i)dx^0 = \underline{\underline{v}}_i dx^i = \underline{\underline{\ell}}(1 - \underline{\underline{\mathbf{w}}}\bullet\mathbf{w})dx^0, \tag{4.17}$$

$$= \underline{\underline{\gamma}}(1 - \underline{\underline{\beta}}\bullet\beta)dx^0 \quad \text{for } \mathbf{u} = \mathbf{0}, \tag{3.33}$$

$$= \underline{\underline{\gamma}}(dx^0 - \underline{\underline{\beta}}\bullet d\mathbf{r}), \tag{3.33}$$

which reduces in the γ–coupling domain, as just shown, to the temporal component of a general Lorentz transformation that was previously derived from $\underline{\underline{w}}d^{1-3}$ of GFT1 for $\mathbf{u} = \mathbf{0}$.

F. The general (1 + 3) EM **w**-deformation of scalar distance measures.[a]

Given (4.15), (4.17), and the (1 + 3) scalar equality $ds = d\underline{s}$, we obtain $(d\underline{r})^2$ in terms of Σ parameters from the equation

$$(d\underline{\mathbf{r}})^2 = (d\underline{x}^0)^2 - (ds)^2 = (\underline{\underline{\beta}}\bullet d\underline{x}^0)^2, \tag{4.18a}$$

$$= (\underline{\underline{v}}_i\,\underline{\underline{v}}_k - g_{ik})dx^i dx^k \equiv m_{ik}\,dx^i dx^k, \tag{4.18b}$$

which evaluates to the expression

$$(d\underline{\mathbf{r}})^2 = [\underline{\underline{\ell}}\,(d\mathbf{r}_{\parallel} - d\underline{\mathbf{r}})]^2 + (d\mathbf{r}_{\perp})^2, \tag{3.17f}$$

previously derived via GFT1 as one expression of $\underline{\underline{w}}d^1$.

G. The general (1 + 3) EM **w**-deformation of scalar GRV's.

Given (4.17) and (4.18) it can be shown that

$$(\underline{\beta})^2 = \frac{[\,(\mathbf{w} - \underline{\underline{\mathbf{w}}})^2 - (\mathbf{w}\times\underline{\underline{\mathbf{w}}})^2]}{(1 - \mathbf{w}\bullet\underline{\underline{\mathbf{w}}}\,)^2}\,, \tag{4.19a}$$

$$= \frac{[\,(\beta - \underline{\underline{\beta}})^2 - (\beta\times\underline{\underline{\beta}})^2]}{(1 - \beta\bullet\underline{\underline{\beta}}\,)^2} \quad \text{for } \mathbf{u} = \mathbf{0}. \tag{4.19b}$$

Giving

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[a] See V.D for the general (1 + 3) EM $\underline{\underline{\mathbf{w}}}$-deformations of arbitrary (1 + 3) tensor quantities at $\underline{O}$ of $\underline{\Sigma}$.

$$\underset{=}{\beta} = \frac{[(\beta_{\|} - \underset{=}{\beta}) - \beta_{\perp}/\underset{=}{\gamma}]}{(1 - \beta \cdot \underset{=}{\beta})} \text{ for } \mathbf{u} = \mathbf{0}, \tag{3.34a}$$

as previously derived via GFT1.

V. Applications and Implications of EM Gravity.

—Contents—

A. The first order equations governing a *free* massive particle (p^{π}) in the $g_{ik}(\Phi^j)$ potentials of EM gravity.

B. The Hamiltonian, Lagrangian, and Action functions, and the Hamilton-Jacobi equations of a free p^{π} particle in EM gravity.

C. The second order dynamics of a p^{π} particle in EM gravity.

D. The EM equations describing how the Φ^i potentials gauge-deform the time and 3-space components of any (1 + 3) tensor so as to insure that the Φ^i, like all other known *field potentials,* are *locally* unobservable.

E. A solution of the photon gravitational-mass/redshift problem.

F. The EM field-equations describing how the $g_{ik}(u^{\alpha})$ potentials are sourced by the (1 + 3) energy-momentum tensor of mass-energy.

G. Some very necessary and fruitful reinterpretations of quantum theory and the experimental data of high-energy particle physics.

A. The first order EM equations of p^{π} particle dynamics.

1. The structures and scalar associated with fundamental EM 4-velocities.

From Eq. (4.15) I obtain the four-scalar differential operator

$$(d/ds) = \ell d/dx^0, \text{ with } \ell \equiv 1/[1 - \mathbf{w}^2]^{1/2}, \tag{5.1}$$

and via this the following first-order 4-velocity vectors and associated scalars:

$$dx^i/ds \equiv \nu^i = (\nu^0, \nu) = \ell\beta^i = \ell(1, \beta) \tag{4.14}$$

$$\nu_i = g_{ik}\nu^i = dx_i/ds = (\nu_0, \nu) = \ell\beta_i = \ell(\beta_0, \underset{\approx}{\beta}) = \ell[(1 - \mathbf{u}^2 + \mathbf{u}\cdot\beta), - \mathbf{w}] \tag{5.2}$$

$$\beta^i\beta_i = \ell^{-2} = (1 - \mathbf{w}^2) \tag{5.3}$$

$$\nu^i\nu_i = \ell^2 \beta^i\beta_i = 1 \tag{5.4}$$

a. Symbols and terminologies for the null limits of **u**, β, *and* **w**.

For $\mathbf{u} \to \mathbf{0}$, $\mathbf{w} \to \beta$ and $\ell \to 1/[1 - \beta^2]^{½} \equiv \gamma$. The environment is the undisturbed 3-space-energy continuum.

For $\beta \to \mathbf{0}$, $\mathbf{w} \to -\mathbf{u}$ and $\ell \to 1/[1 - \mathbf{u}^2]^{½} \equiv \Gamma$. The p^π particle is immersed in the potentials $\Phi^i(x^i)$ while remaining at rest relative to Σ.

If $\mathbf{w} \to \mathbf{0}$ because $\beta = \mathbf{u} \to \mathbf{0}$: $\ell \to 1$ The p^π particle is in its 3-space-energy *ground state* of *absolute rest in* ${}^0\mathcal{E}_m$.

If $\mathbf{w} \to \mathbf{0}$ because $\beta = -\mathbf{u}$: $\ell \to 1$ The p^π particle is immersed in the potentials $\Phi^i(x^i)$ but is in a state of *convective free fall* or *relative absolute rest.*

2. The EM energy-momentum 4-vector p^i.

Converting momentum units to energy-units via an implicit factor of lightspeed, and defining $E^0 = m^0c^2$, the EM energy-momentum 4–vector has the following alternative forms:

$$p^i = (p^0, \mathbf{p}) = E^0\nu^i = (\ell E^0, \ell E^0\beta) = (E, E\beta) = E\beta^i. \tag{5.5}$$

For $\mathbf{u} = \mathbf{0}$, $\ell \to \gamma$, and p^i attains the special relativistic form. Hence, with p^π momentum conserved in the form prescribed by p^i, and with $\mathbf{F} = d\mathbf{p}/dx^0$ as an accurate description of the Newtonian force, we see that the equivalence of *mass and energy*, and the variation of *mass-energy* with $\delta\mathcal{E}_m$ propagation velocity, as previously deduced via GFT1 and described by

$$\underline{\underline{w}}d^4 = \underline{\underline{w}}d^5$$

$$mc^2/m^0c^2 = E/E^0 = \ell = 1/[1 - \mathbf{w}^2]^{½}\,, \tag{5.6}$$

can also be regarded as an immediate consequences of the expanded dynamics and the larger $\in_m$ physics underlying EM gravity that results from Maxwell's gravity theorem.

3. The EM Hamiltonian-momentum 4-vector p_i.

From Eqs. (4.14,15) I derive the following structures and interpretations of the Hamiltonian-momentum 4-vector:

$$p_i = g_{ik}p^k = (p_0, \vec{\mathbf{p}}) = E^0\nu_i = E\beta_i = [E(1 + \mathbf{u}\bullet\mathbf{w}), -\mathbf{w}E] = (H, -\mathbf{w}E), \text{ where} \tag{5.7}$$

$$H = E\beta_0 = E(1 - \mathbf{u}^2 + \mathbf{u}\bullet\beta) = (E - \vec{\mathbf{p}}\bullet\mathbf{u}) = [E(1 - \mathbf{u}^2) + \mathbf{p}\bullet\mathbf{u}], \tag{5.8}$$

describes the Hamiltonian of a p^π particle in the g_{ik} potentials of EM gravity.

a. Terminologies for the null limits of **u**, β, and **w**.

The total energy of a p^π particle is:

For $\mathbf{u} = \mathbf{0}$, $H(u = 0, \beta = \mathbf{w}) = \gamma E^0 = E^0/[1 - \beta^2]^{½}$.	Above its ground state (rest) value.	(5.9)
For $\beta = \mathbf{0}$, $H(\beta = 0, \mathbf{w} = -\mathbf{u}) = E^0/\Gamma = (1 - \mathbf{u}^2)E$.	Below its ground-state (rest) value.	(5.10)
For $\mathbf{w} = \mathbf{0}$, $H(\beta = \mathbf{u}) = E^0$ (i.e., abs. or rel. free fall).	Equal to its ground state (rest) value.	(5.11)

The phrase *at rest in EM gravity* will always be taken to mean *in the sense that* $\beta = \mathbf{0}$, and thus, $\mathbf{w} = -\,\mathbf{u}$. Hence, if a p^{π} particle is at rest in EM gravity, it would take $(\Gamma - 1)H = E^0(1 - \Gamma^{-1})$ joules of work to pull it free of gravity and thus *up* to its ground-state energy E^0.

b. The EM energy-Hamiltonian momentum scalar.

The 4-scalar associated with p^{π} energy and momentum in a Φ^i field has the following forms:

$$p^i p_i = [\,(p^0)^2 - \vec{\mathbf{p}}^{\,2}\,] = [\,E^2 - \vec{\mathbf{p}}^{\,2}\,] = (\,EH + \mathbf{p}\bullet\vec{\mathbf{p}}\,) = (E^0)^2. \tag{5.12}$$

B. The Hamiltonian, Lagrangian, and Action functions, and the Hamilton Jacobi Equations of a p^{π} particle in the Φ^i potentials of EM gravity.

The following information may be of some use to theorists interested in formulating the *quantum characteristics* of EM gravity.

1. Consistent expressions and relations for Action.

As with p^{π} momentum, I here use a lightspeed scaled action $S \equiv cS$. Hence, p^i, p_i, H, and the Lagrangian L, will consistently denote *mass-energy* while S denotes *mass-energy-length* or *mass-energy-distance*. From the perspective of Σ the quantities S, L, ds, and $p_i dx^i$ are interrelated as follows:

$$S = \int dS \equiv \int L dx^0 \qquad = -\int E^0 ds \quad = -\int p_i v^i ds = -\int p_i dx^i = -\int [p_0 dx^0 + \vec{\mathbf{p}}\bullet d\mathbf{r}], \tag{5.13a}$$

$$= -\int (E^0/\ell) dx^0 \;= -\int [p_0 + \vec{\mathbf{p}}\bullet \beta] dx^0 \;=\; -\int (p_i\beta^i) dx^0 = -E^0 \int d\underline{x}^0, \tag{5.13b}$$

$$= \quad \int (\partial_i S) dx^i \quad = \quad \int [\,(\partial_0 S) dx^0 + (\nabla S)\bullet d\mathbf{r}] = \; -\int p_i dx^i. \tag{5.13c}$$

Equations (5.13) contain only three distinct integrands: A self-definition of differential *Action,* dS. The historic representation of dS as $L dx^0$. And five mathematically equivalent expressions of the *differential Action* 4-scalar, $dS = -\,p_i dx^i$, which is a (1 + 3) tensor generalization of the historically earliest Action integrand, $m^0 c\beta\bullet d\mathbf{r}$, associated with the principle of least action.

2. Lagrangian and Hamiltonian relations unique to EM gravity.

From Eqs. (5.13), I obtain the following equivalent expressions for the Lagrangian of a p^{π} particle in the g_{ik} potentials of EM gravity at x^i of Σ^d:

$$L = -\,E^0\,[1 - (\beta - \mathbf{u})^2\,]^{1/2} = -\,(p_0 - \mathfrak{p}\bullet\beta) \;= -(H - \mathfrak{p}\bullet\beta) = -\,p_i\beta^i = -\,p^i\beta_i, \tag{5.14}$$

where I have introduced the symbol, $\mathfrak{p} \equiv -\vec{\mathbf{p}}$, to denote Hamilton's *generalized momentum* (scaled by c to convert it to energy units). Hamilton's generalized momentum then has the following definition and meaning for EM gravity:

$$\mathfrak{p} \;= \partial L/\partial\beta \;=\; \mathbf{w}E \;=\; -\;\vec{\mathbf{p}} \;=\; \mathbf{p} - \mathbf{u}E \;=\; \nabla S, \tag{5.15}$$

with,

$$p_0 = \;-\,\partial S/\partial x^0\,, \tag{5.16a}$$

$$\vec{\mathbf{p}} \;=\; -\,\nabla S. \tag{5.16b}$$

3. Hamilton's function and the Hamiltonian for EM gravity.

Eq. (5.15) shows that $\mathfrak{p}$ is generally different from both $\vec{\mathbf{p}}$ and **p**. These definitions and relations, stemming directly from Eqs. (5.13), are consistent with Hamilton's function:

$$H(\mathfrak{p},\mathbf{u}) = \mathfrak{p}\bullet\beta - L(\beta,\mathbf{u}), \tag{5.17}$$

$$= E^0[(1- \mathbf{u}^2) + \mathbf{u}\bullet\beta]/(1 - \mathbf{w}^2), \tag{5.18}$$

for converting $L(\beta,\mathbf{u})$ into $H(\mathfrak{p}, \mathbf{u})$, and for generating the partial differential *Hamiltonian-Jacobi* [Action] equation, symbolically denoted by

$$H(\nabla S, x^i) + \partial S/\partial x^0 = 0. \tag{5.19}$$

Comparing (5.19) to (5.16a), we recover the well-known fact that $p_0 = H$. That is, the temporal component of the covariant 4-energy-momentum vector contains the total energy of a p^π particle. Thus it is appropriate to call p_i the *Hamiltonian-momentum 4-vector*. We then both confirm and derive that the equations

$$H(\mathfrak{p}, \mathbf{u}) = p_0 = [(\mathfrak{p})^2 + (E^0)^2\,]^{1/2} + \mathbf{u}\bullet\mathfrak{p}\,, \tag{5.20a}$$

$$= \qquad E \qquad + \mathbf{u}\bullet\mathfrak{p}, \tag{5.20b}$$

describe the standard and the non standard *Hamiltonians* for a p^π particle in the field of EM gravity at x^i of Σ^d, with the non standard expression, (5.20b), illuminating the fact that the difference between H and E is driven by $\mathbf{u}\bullet\mathfrak{p}$.

4. The Hamilton-Jacobi equation for p^π and p^0 particles in EM gravity.

Using $\partial_i \equiv \partial/\partial x^i$, the *Hamilton-Jacobi* equation takes the explicit forms:

$$[\,(\nabla S)^2 + g^{ik}(\partial_i S)\,(\partial_k S)\,]^{1/2} + \mathbf{u}\bullet\nabla S + \partial_0 S = 0, \tag{5.21}$$

$$[\mathbf{u}\bullet\nabla S + \partial_0 S\,]^2 - (\nabla S)^2 - g^{ik}(\partial_i S)\,(\partial_k S) = 0, \tag{5.22}$$

where,
$$g^{ik}(\partial_i S)\,(\partial_k S) = g^{ik}p_i p_k = p^i p_i = (E^0)^2. \tag{5.23}$$

These equations govern the temporal evolution of the Action of a p particle in the gauge field of Einstein-Maxwell gravity at x^i of Σ^d. For a p^0 particle, $E^0 = 0$, and they reduce to:

$$|\nabla S| + \mathbf{u}\bullet(\nabla S) + \partial S/\partial x^0 = 0. \tag{5.24}$$

5. Hamilton's canonical equation f motion for $\beta(p)$ in the field of EM gravity.

Using Hamilton's canonical equation of motion for the 3-velocity of a p particle,

$$\beta(p) = \partial H/\partial \mathfrak{p}, \tag{5.25}$$

and Eqs. (5.20), I obtain:

$$\beta(p) = \partial H/\partial \mathfrak{p} = \mathfrak{p} /[(\mathfrak{p})^2 + (E^0)^2]^{1/2} + \mathbf{u},$$

$$= \mathfrak{p}/E + \mathbf{u}, \tag{5.26}$$

$$= \mathbf{w} + \mathbf{u} \text{ for } E^0 > 0, \tag{2.3}$$

$$= \mathbf{w}^0 + \mathbf{u} \text{ for } E^0 = 0, \tag{2.3}$$

thereby recovering the cornerstone GRV equation (2.3) of hyper energy theory.

6. On the small w convergence of H to its pre-relativistic form; $T + V_H$.

Expanding (5.8) and retaining only second order terms in **u** and β, we obtain $H = T + V_H$ where,

$$T = \tfrac{1}{2}E^0 \beta^2, \tag{5.27}$$

$$V_H = -E^0(\tfrac{1}{2}\mathbf{u}^2) + E^0, \tag{5.28a}$$

$$\equiv -E^0\Phi_m + E^0. \tag{5.28b}$$

Since the constant E^0 term in V_H does not affect interaction forces, we see that in this small w approximation the Hamiltonian for a p^π particle of rest energy E^0 in an EM gravity field agrees with the pre relativistic formulation of H, provided that $(-\tfrac{1}{2}\mathbf{u}^2)E^0$ is the hyper energy explanation for the Newtonian potential, Φ_n, in accordance with the previous deduction

$$\Phi_m = \tfrac{1}{2}\mathbf{u}^2 = -\Phi_n/c^2, \tag{3.80a}$$

from Maxwell's Gravity theorem.

Since this result follows simply from $ds = dx^0/\ell$ and the identification of p_0 as H, it offers yet another example of the extreme effectiveness of the (1 + 3) tensor calculus.

8. On the small w convergence of L to its pre relativistic form; $T - V_L$.

Expanding (5.14) and retaining only second order terms in **u** and β, we obtain $L = T - V_L$, where

$$T = \tfrac{1}{2}E^0 \beta^2, \tag{5.29}$$

$$V_L - E^0 = -E^0(\Phi_m - \mathbf{u}\cdot\beta), \tag{5.30}$$

$$= -E^0\Phi_i\beta^i, \tag{5.31}$$

where,

$$\Phi_i = (\Phi_m, \vec{\Phi}) = (\Phi_m, -\mathbf{u}). \tag{5.32}$$

Since the constant E^0 term in V_L does not affect the gravitational interaction, and since it is understood that $-E^0\Phi_m$ is the stationary gravitational mass defect, we see that, in this small w approximation, the Lagrangian for E^0 in an EM gravity field contains the pre relativistic formulation plus a field-GRV coupling term which gives the gravitational interaction potential, V_L, the same structure as the electromagnetic interaction potential, L_e, in a Lagrangian for a charge e in an electromagnetic field possessing a covariant 4-potential, $\varphi_i = -(\varphi_e, -c\mathbf{A})$. Namely,

$$L_e = -e\varphi_i\beta^i = -e(\varphi_e, -c\mathbf{A}\bullet\beta). \qquad (5.33)$$

Thus, although $\Phi_i = (\Phi_m, -\mathbf{u})$ is not a covariant 4-vector, it is nevertheless functionally identical to the covariant electromagnetic 4-potential $\varphi_i = (\varphi_e, -c\mathbf{A})$, and as such it provide additional assurance that EM gravity can be readily quantized—to predict the kinds of $\delta\mathcal{E}_m$ particles that can be precipitated specifically by collisionally unleashed gravitational mass-energy, as apposed to collisionally unleashed kinetic mass-energy.

C. The second order dynamics of a free p^π particle in EM gravity.

1. The contravariant power and force equations.

With $g_{ik}(u^\alpha(x^i))$ and $g^{ik}(u^\alpha(x^i))$ defined by Eqs. (4.13), and $p^i(x^i)$ defined by Eqs. (5.5-6), we now utilize the well known (1 + 3) tensor equations:

$$Dp^i/ds = dp^i/ds + \Gamma^i{}_{jk}\, p^j\nu^k = \mathbf{F}^{\mathbf{i}}(x^i), \qquad (5.34)$$

where,

$$\Gamma^i{}_{jk}(x^i) = \Gamma^i{}_{kj} = g^{im}[\,\partial g_{jm}/\partial x^k + \partial g_{mk}/\partial x^j - \partial g_{jk}/\partial x^m\,], \qquad (5.35)$$

to describe, at x^i of Σ^d, the contravariant dynamics of a p^π particle in a general $g_{ik}(u^\alpha(x^i))$ field of EM gravity in response to a *non-gravitational* 4-force, $\mathbf{F}^{\mathbf{i}}$, subject to the tentative *free field-particle* restriction defined by $\mathbf{F}^{\mathbf{i}} = 0$. The contravariant equation of motion for a free p^π particle in an EM gravity field is then:

$$dp^i/ds = -\Gamma^i{}_{jk}\, p^j\nu^k. \qquad (5.36)$$

a. The 4-rate of change of contravariant p^π energy, dp^0/ds.

Using the components of $\Gamma^0{}_{jk}(x^i)$ listed in Appendix A, and $p^0 = E$, we obtain:

$$dp^0/ds = \ell dp^0/dx^0 = -\Gamma^0{}_{jk}p^j\nu^k = E\ell\psi, \qquad (5.37)$$

where,

$$\psi = -\mathbf{w}\bullet[(\mathbf{w}\bullet\nabla)\mathbf{u}]. \qquad (5.38)$$

Hence

$$d[\ln(E/E^0)]/dx^0 = [d(\tfrac{1}{2}\mathbf{w}^2)/dx^0]/(1 - \mathbf{w}^2) = -\mathbf{w}\bullet[(\mathbf{w}\bullet\nabla)\mathbf{u}] = \psi. \qquad (5.39)$$

*b. The 4-rate of change of contravariant p^π momentum, d**p**/ds.*

Using the components of $\Gamma^\alpha{}_{jk}(x^i)$ listed in Appendix A, $\Gamma_{jk}(x^i) = \mathbf{e}_\alpha\Gamma^\alpha{}_{jk}(x^i)$, and $\mathbf{p} = \beta E$, we obtain:

$$d\mathbf{p}/ds = \ell d\mathbf{p}/dx^0 = -\,\Gamma_{jk}(x^i)p^j v^k, \tag{5.40}$$

$$d\mathbf{p}/dx^0 = E\,[\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \beta\times(\nabla\times\mathbf{u}) + \psi\mathbf{u}\,]. \tag{5.41}$$

And inserting Eq. (5.39) into Eq. (5.41), we then obtain the Σ–acceleration of a free p^π particle in a general $g_{ik}(u^\alpha(x^i))$ field of EM gravity as:

$$d\beta/dx^0 = d^2\mathbf{r}/(dx^0)^2 = \mathbf{a}(x^i) = [\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \beta\times(\nabla\times\mathbf{u}) - \psi\mathbf{w}\,], \tag{5.42}$$

$$= [\,d\mathbf{u}/dx^0 - \mathbf{w}\times(\nabla\times\mathbf{u}) - \psi\mathbf{w}\,], \tag{5.43}$$

where

$$d\mathbf{u}/dx^0 = \partial\mathbf{u}/\partial x^0 + (\beta\cdot\nabla)\mathbf{u}, \tag{5.44a}$$

$$= \partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \mathbf{u}\times(\nabla\times\mathbf{u}). \tag{5.44b}$$

From (5.43) it is clear that, with respect to Σ, the acceleration of a p^π particle in *free fall* (**w** = 0) reduces to the well-known expression (5.44) for the total 3-acceleration of a fluid medium. And from (5.38) and (5.42) we find that p^π particle which is instantaneously at rest at x^i of Σ^d will initially accelerate at the rate

$$d\beta/dx^0 = d^2\mathbf{r}/(dx^0)^2 = \mathbf{a}(x^i) = [\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \mathbf{u}(\mathbf{u}\cdot\nabla\Phi_m)\,]. \tag{5.45}$$

c. The remarkable similarity of electrodynamic and gravodynamic forces.

Consider the pair of equations:

$$d(\ell E^0\beta)/dx^0 = \mathbf{f}_{Em} = E\,[\,\partial\mathbf{u}/\partial x^0 + \nabla\Phi_m - \beta\times(\nabla\times\mathbf{u}) + (\psi)\mathbf{u}\,]. \tag{5.41}$$

$$d(\gamma E^0\beta)/dx^0 = f_L = cQ_-[\,\partial\mathbf{A}/\partial x^0 + \nabla(\varphi/c) - \beta\times(\nabla\times\mathbf{A}) + (0)\mathbf{A}\,]. \tag{5.46}$$

The first equation describes the generally covariant *EM gravity force* ($\mathbf{f}_{Em}$) on a p^π particle with rest energy E^0 which is *moving* with a GRV, $\beta = \mathbf{u} + \mathbf{w}$, through the *acceleration-field potentials* $g_{ik}(u^\alpha)$ of EM gravity at x^i of Σ^d. While (5.46) describes the covariant *Lorentz force* ($\mathbf{f}_L$) on a p^π particle with rest energy E^0 and negative electric charge Q_- which is *propagating* with GRV, $\beta = \mathbf{w}$, through the electromagnetic potentials, $\varphi_e^i(x^i) = (\varphi_e, \mathbf{A}^*)$, at x^i of Σ^d where $g_{ik} = G_{ik}$ = diagonal (1, −1, −1, −1).

Since electromagnetism is readily quantizable, this remarkable similarity provides further assurance of the quantizability of EM gravity.

2. The covariant power and force equations.

With $g_{ik}(u^{\alpha}(x^i))$ defined by Eqs. (4.13), and $p_i(x^i)$ defined by Eqs. (5.7-8), we now utilize the well-known (1 + 3) tensor equations:

$$Dp_i/ds = dp_i/ds - \tfrac{1}{2}(\partial g_{jk}/\partial x^i)\, p^j \nu^k = \mathbf{F_i}(x^i) = 0, \tag{5.47}$$

to describe, at x^i of Σ^d, the covariant dynamics of a p^{π} particle in a general $g_{ik}(u^{\alpha}(x^i))$ field of EM gravity in response to a *non-gravitational* 4-force, $\mathbf{F_i}$, subject to the tentative *free field-particle* restriction defined by $\mathbf{F_i} = 0$. The covariant equation of motion for a free p^{π} particle in an EM gravity field is then:

$$dp_i/ds = \tfrac{1}{2}(\partial g_{jk}/\partial x^i)\, p^j \nu^k. \tag{5.48}$$

a. The 4-rate of change of total p^{π} energy, dp_0/ds.

Using $p_0 = H$, and $\partial_0 = \partial/\partial x^0$, the 4-rate of change of the total energy of a free p^{π} particle in an EM gravity field is described by

$$dp_0/ds = dH/ds = \ell dH/dx^0 = \tfrac{1}{2}\, \partial_0[g_{jk}(u^{\alpha})]\, p^j \nu^k = \tfrac{1}{2}\ell E\, \partial_0[g_{jk}(u^{\alpha})]\, \beta^j \beta^k, \tag{5.49a}$$

$$= \ell E\, \mathbf{w}\bullet(\partial \mathbf{u}/\partial x^0), \tag{5.49b}$$

or,

$$dH/dx^0(x^i) = E^0\, \mathbf{w}\bullet(\partial \mathbf{u}/\partial x^0)/[1 - \mathbf{w}^2]^{1/2}, \tag{5.49c}$$

$$= -E^0\, \Gamma(\partial \Phi_m/\partial x^0), \text{ (for } \beta = \mathbf{0}). \tag{5.49d}$$

b. The 4-rate of change of covariant p^{π} momentum, $d\vec{\mathbf{p}}/ds$.

The 4-rate of change of the covariant component of p^{π} momentum is then given by

$$d\vec{\mathbf{p}}/d\mathbf{s} = \ell dp_{\gamma}/dx^0 = \tfrac{1}{2}(\partial g_{jk}/\partial x^{\gamma})\, p^j \nu^k = \tfrac{1}{2}\ell E\, \partial_{\gamma}[g_{jk}(u^{\alpha})]\, \beta^j \beta^k, \tag{5.50a}$$

$$= E\, [-\nabla \Phi_m + \beta \times (\nabla \times \mathbf{u}) + \beta \bullet \nabla)\mathbf{u}]. \tag{5.50b}$$

D. The general (1 + 3) EM **w**-deformations of (1 + 3) tensor quantities.

As in IV.E, we let $\underline{v}^i = \underline{\ell}\underline{\beta}^i$ be the 4–GRV of both $\underline{O}$ and $\underline{\mathbf{r}}$, so that the reference point $`\underline{\mathbf{r}}$ and a $\delta\mathcal{E}_m$ particle at x^i of Σ^d are embedded in the same $\in_m$ flow field, $\underline{\mathbf{u}} = \mathbf{u}$. Hence, $\underline{\mathbf{w}}$ and **w** are thus consistently defined by $\underline{\mathbf{w}} = \underline{\beta} - \mathbf{u}$, and $\mathbf{w} = \beta - \mathbf{u}$.

The temporal component of a general Lorentz transformation,

$$d\underline{x}^0 = \underline{v}_i dx^i = \underline{\ell}[(1 + \mathbf{u}\bullet\underline{\mathbf{w}}), -\underline{\mathbf{w}}]\bullet(dx^0, d\mathbf{r}), \quad (4.17)$$

$$= \underline{\ell}(1 - \mathbf{w}\bullet\underline{\mathbf{w}})dx^0, \quad (4.17)$$

may then be regarded as the temporal component of the (1 + 3) tensor equation,

$$d\underline{x}^i(\underline{\beta}, \mathbf{u}, dx^k) = E^i{}_k(\underline{\beta}, \mathbf{u})dx^k, \quad (5.51)$$

which describes the degree to which the numbers, $d\underline{x}^i$, that quantify contravariant tensor elements in $\underline{\Sigma}$ are unwittingly *inflated* or *deflated* relative to the dx^k, due to the fact that the relevant measurement standards in $\underline{\Sigma}$ are unknowingly w-deformed.

For example. Take the case of an oscillator which is at rest at $\underline{O}$ of $\underline{\Sigma}$ in the Γ-coupling domain. Then $d\underline{x}^0 = \Gamma(1 - \mathbf{u}^2)dx^0 = dx^0/\Gamma$ shows that the number obtained in $\underline{\Sigma}$ for the period of the oscillator is unwittingly *deflated,* due to $\underline{w}d^2$.

It follows that the inverse equation

$$dx^i(\underline{\beta}, \mathbf{u}, d\underline{x}^k) = D^i{}_k(\underline{\beta}, \mathbf{u})d\underline{x}^k, \quad (5.52)$$

describes how the $d\underline{x}^i$ at $\underline{O}$ in $\underline{\Sigma}$—regarded as empirically quantified proper quantities—are gauge deformed by $\underline{\beta}$ and/or **u** so as to preserve the Null-$\underline{\mathbf{p}}_m$ Axiom, thereby causing $\underline{\Sigma}^\ell$ to be empirically equivalent to Σ^ℓ. Using the previous example, $dx^0 = \Gamma d\underline{x}^0$, or $T = \Gamma\, \underline{T}^0 = \Gamma\, T^0$, shows that, from the perspective of Σ, the period of an oscillator at rest in $\underline{\Sigma}$ is increased by the Γ-coupling in accordance with $\underline{w}d^2$. And since $dx^i dx_i = d\underline{x}^k d\underline{x}_k = ds^2$ it follow that $[D] = [E]^{-1}$ and/or $D^i{}_m D^m{}_k = \delta^i{}_k$.

It follows that the $D^i{}_k$ contain $\underline{w}d^{1-3}$ in a form that applies specifically to the elements of a contravariant 4-vector, for arbitrary $\underline{\mathbf{w}}$ and **u**. And it then follows that the $D^i{}_k$ deformations must apply equally well to *any contravariant vector*, $\underline{A}^i$. And since $\underline{A}^i\underline{A}_i = A^iA_i$, we must conclude that whereas *contravariant* 4-vectors are **w**-deformed by

$$A^i(\underline{\beta}, \mathbf{u}, \underline{A}^k) = D^i{}_k(\underline{\beta}, \mathbf{u})\underline{A}^k, \quad (5.53a)$$

covariant 4-vectors are **w**-deformed by

$$A_i(\underline{\beta}, \mathbf{u}, \underline{A}_k) = E^k{}_i(\underline{\beta}, \mathbf{u})\underline{A}_k. \quad (5.54)$$

Given $E^i{}_k(\underline{\beta}, \mathbf{u})$ and $D^i{}_k(\underline{\beta}, \mathbf{u})$ one can then obtain the more complicated **w-**deformation of a (1 + 3) tensor of arbitrary mixed rank in accordance with the rule indicated by

$$T_{ik} = E^\ell{}_i E^m{}_k \underline{T}_{\ell m}, \quad (5.55)$$

$$T^{ik} = D^i{}_\ell D^k{}_m \underline{T}^{\ell m}. \quad (5.56)$$

And we use this rule to actually derive $E^i{}_k$ and $D^i{}_k$ in accordance with the following expanded form of the Null-$\underline{\mathbf{p}}_m$ Axiom.

1. The expanded Null-$\underline{\mathbf{p}}_m$ Axiom.

The expanded Null-$\underline{\mathbf{p}}_m$ Axiom states that:

The $g_{ik}(u^\alpha)$ at $\underline{x}^i$ of Σ^d must be compatible with
$g_{ik} = g^{ik} \equiv G_{ik} = \text{diagonal }(1, -1,-1,-1)$ at $\underline{O}$ of $\underline{\Sigma}$.

The tensor E^i_k (and thus D^i_k) can then be derived from the equations

$$g_{ik}(u^\alpha) = E^\ell_i E^m_k\, G_{\ell m}, \tag{5.57}$$

$$g^{ik}(u^\alpha) = D^\ell_i D^m_k\, G^{\ell m}, \tag{5.58}$$

by using equations (4.13) for $g_{ik}(u^\alpha)$ and $g^{ik}(u^\alpha)$, and

$$E^0_i(\underline{\underline{\beta}}, \mathbf{u}) = \underline{\underline{v}}_i, \tag{5.59}$$

from (4.17) and (5.51). The derivations are carried out in Appendix B and the results are displayed as follows.

2. Contravariant 4-vectors are gauge deformed by:

$$D^i_k = \begin{array}{c|cccc} i \backslash j\to & 0 & 1 & 2 & 3 \\ \hline 0 & \underline{\underline{\ell}} & \underline{\underline{\ell}}\,\underline{\underline{w}}_1 & \underline{\underline{\ell}}\,\underline{\underline{w}}_2 & \underline{\underline{\ell}}\,\underline{\underline{w}}_3 \\ 1 & \underline{\underline{\ell}}\,\underline{\underline{\beta}}_1 & 1+[g\underline{\underline{w}}_1+\underline{\underline{\ell}}u_1]\underline{\underline{w}}_1 & [g\underline{\underline{w}}_1+\underline{\underline{\ell}}u_1]\underline{\underline{w}}_2 & [g\underline{\underline{w}}_1+\underline{\underline{\ell}}u_1]\underline{\underline{w}}_3 \\ 2 & \underline{\underline{\ell}}\,\underline{\underline{\beta}}_2 & [g\underline{\underline{w}}_2+\underline{\underline{\ell}}u_2]\underline{\underline{w}}_1 & 1+[g\underline{\underline{w}}_2+\underline{\underline{\ell}}u_2]\underline{\underline{w}}_2 & [g\underline{\underline{w}}_2+\underline{\underline{\ell}}u_2]\underline{\underline{w}}_3 \\ 3 & \underline{\underline{\ell}}\,\underline{\underline{\beta}}_3 & [g\underline{\underline{w}}_3+\underline{\underline{\ell}}u_3]\underline{\underline{w}}_1 & [g\underline{\underline{w}}_3+\underline{\underline{\ell}}u_3]\underline{\underline{w}}_2 & 1+[g\underline{\underline{w}}_3+\underline{\underline{\ell}}u_3]\underline{\underline{w}}_3 \end{array}, \tag{5.60}$$

where

$$g = g(\underline{\underline{\ell}}) = \underline{\underline{\ell}}^2/(1+\underline{\underline{\ell}}) = \underline{\underline{\ell}}\, f(\underline{\underline{\ell}}), \tag{5.61}$$

a. The elements and limits of $D^i_k(\underline{\underline{\beta}}, \mathbf{u})$.

$D^0_k = \underline{\underline{\ell}}(1, \underline{\underline{w}}^\alpha)$, (5.62a)

$= \underline{\underline{\gamma}}(1, \underline{\underline{\beta}}^\alpha)$ in the γ-coupling domain, (5.62b)

$= \Gamma(1, -u^\alpha)$ in the Γ-coupling domain. (5.62c)

$D^\alpha_0 = \underline{\underline{\ell}}\,\underline{\underline{\beta}}^\alpha \neq D^0_\alpha$ (unless $\mathbf{u} = \mathbf{0}$). (5.63a)

$= \underline{\underline{\gamma\beta}}^\alpha$ in the γ-coupling domain, (5.63b)

$= 0$ in the Γ-coupling domain. (5.63c)

$D^\alpha_\upsilon = \delta^\alpha_\upsilon + [g\, \underline{\underline{w}}^\alpha + \underline{\underline{\ell}}\, u^\alpha]\underline{\underline{w}}^\upsilon$, (5.64a)

$= \delta^\alpha_\upsilon + g(\underline{\underline{\gamma}})\underline{\underline{\beta}}^\alpha\underline{\underline{\beta}}^\upsilon$ in the γ-coupling domain, (5.64b)

$= \delta^\alpha_\upsilon - f(\Gamma)u^\alpha u^\upsilon$ in the Γ-coupling domain (see (5.61) for f). (5.64c) *b. The γ-coupling and Γ-coupling domain limits of* $D^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

In the γ-coupling domain with:

$$g = g(\underline{\underline{\gamma}}) = \underline{\underline{\gamma}}^2/(1 + \underline{\underline{\gamma}}) \qquad \text{and} \qquad \underline{\underline{\beta}}^\alpha = \underline{\underline{\beta}}^1 = \underline{\underline{\beta}}.$$

$$D^i{}_k = \begin{array}{c|cccc} i \backslash j\rightarrow & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \hline \mathbf{0} & \underline{\underline{\gamma}} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 \\ \mathbf{1} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & 1+g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_3 \\ \mathbf{2} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_1 & 1+g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_3 \\ \mathbf{3} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_2 & 1+g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_3 \end{array} \rightarrow \begin{array}{c|cccc} i \backslash j\rightarrow & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \hline \mathbf{0} & \underline{\underline{\gamma}} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}} & 0 & 0 \\ \mathbf{1} & \underline{\underline{\gamma}}\,\underline{\underline{\beta}} & \underline{\underline{\gamma}} & 0 & 0 \\ \mathbf{2} & 0 & 0 & 1 & 0 \\ \mathbf{3} & 0 & 0 & 0 & 1 \end{array}. \qquad (5.65)$$

And in the Γ-coupling domain with:

$$f = f(\Gamma) = \Gamma/(1 + \Gamma) \qquad \text{and} \qquad \text{and } u^\alpha = u^1 = u.$$

$$D^i{}_k = \begin{array}{c|cccc} i \backslash j\rightarrow & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \hline \mathbf{0} & \Gamma & -\Gamma u_1 & -\Gamma u_2 & -\Gamma u_3 \\ \mathbf{1} & 0 & 1 - f u_1^2 & -f u_1 u_2 & -f u_1 u_3 \\ \mathbf{2} & 0 & -f u_2 u_1 & 1 - f u_2^2 & -f u_2 u_3 \\ \mathbf{3} & 0 & -f u_3 u_1 & -f u_3 u_2 & 1 - f u_3^2 \end{array} \rightarrow \begin{array}{c|cccc} i \backslash j\rightarrow & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \hline \mathbf{0} & \Gamma & -\Gamma u & 0 & 0 \\ \mathbf{1} & 0 & \Gamma^{-1} & 0 & 0 \\ \mathbf{2} & 0 & 0 & 1 & 0 \\ \mathbf{3} & 0 & 0 & 0 & 1 \end{array}. \qquad (5.66)$$

3. Covariant 4-vectors are then **w**-gauge deformed by:

$$E^i{}_k = \begin{array}{c|cccc} i \backslash j\rightarrow & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \hline \mathbf{0} & \underline{\underline{\boldsymbol{\ell}}}(1 + \mathbf{u}\bullet\underline{\underline{\mathbf{w}}}) & -\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_1 & -\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_2 & -\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_3 \\ \mathbf{1} & -(\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_1 + G_1) & 1 + g\,\underline{\underline{w}}_1^2 & g\,\underline{\underline{w}}_1\,\underline{\underline{w}}_2 & g\,\underline{\underline{w}}_1\,\underline{\underline{w}}_3 \\ \mathbf{2} & -(\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_2 + G_2) & g\,\underline{\underline{w}}_2\,\underline{\underline{w}}_1 & 1 + g\,\underline{\underline{w}}_2^2 & g\,\underline{\underline{w}}_2\,\underline{\underline{w}}_3 \\ \mathbf{3} & -(\underline{\underline{\boldsymbol{\ell}}}\,\underline{\underline{w}}_3 + G_3) & g\,\underline{\underline{w}}_3\,\underline{\underline{w}}_1 & g\,\underline{\underline{w}}_3\,\underline{\underline{w}}_2 & 1 + g\,\underline{\underline{w}}_3^2 \end{array}, \qquad (5.67)$$

where

$$g = g(\underline{\underline{\ell}}) = \underline{\underline{\ell}}^2/(1 + \underline{\underline{\ell}}) = \underline{\underline{\ell}}\, f(\underline{\underline{\ell}}). \qquad (5.68)$$

$$\mathbf{G} = \{\, g[\underline{\underline{\mathbf{w}}}\times(\underline{\underline{\mathbf{w}}}\times\mathbf{u}) + \underline{\underline{\ell}}\mathbf{u} \,\}, \qquad (5.69a)$$

$$= \{\, g(\mathbf{u}\bullet\underline{\underline{\mathbf{w}}})\underline{\underline{\mathbf{w}}} + \mathbf{u} \,\}. \qquad (5.69b)$$

a. The elements and limits of $E^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

$$E^0{}_k = \underline{\underline{\ell}}[(1 + \mathbf{u}\bullet\underline{\underline{\mathbf{w}}}), -\underline{\underline{w}}^\alpha] \tag{5.70a}$$

$$= \underline{\underline{\gamma}}(1, -\underline{\underline{\beta}}^\alpha) \text{ in the } \gamma\text{-coupling domain,} \tag{5.70b}$$

$$= \Gamma[(1 - \mathbf{u}^2), u^\alpha) \text{ in the } \Gamma\text{-coupling domain.} \tag{5.70c}$$

$$E^\alpha{}_0 = -\{\underline{\underline{\ell}}\ \underline{\underline{w}}^\alpha + g(\mathbf{u}\bullet\underline{\underline{\mathbf{w}}})\underline{\underline{w}}^\alpha + u^\alpha\} \neq E^0{}_\alpha \text{ (unless } \mathbf{G} = \mathbf{0}). \tag{5.71a}$$

$$= -\underline{\underline{\gamma}}\underline{\underline{\beta}}^\alpha \text{ in the } \gamma\text{-coupling domain,} \tag{5.71b}$$

$$= 0 \quad \text{in the } \Gamma\text{-coupling domain.} \tag{5.71c}$$

$$E^\alpha{}_\upsilon = \delta^\alpha{}_\upsilon + g\ \underline{\underline{w}}^\alpha\underline{\underline{w}}^\upsilon, \tag{5.72a}$$

$$= \delta^\alpha{}_\upsilon + g(\underline{\underline{\gamma}})\underline{\underline{\beta}}^\alpha\underline{\underline{\beta}}^\upsilon \text{ in the } \gamma\text{-coupling domain,} \tag{5.72b}$$

$$= \delta^\alpha{}_\upsilon + g(\Gamma)u^\alpha u^\upsilon \text{ in the } \Gamma\text{-coupling domain.} \tag{5.72c}$$

b. The γ*-coupling and* Γ*-coupling domain limits of* $E^i{}_k(\underline{\underline{\beta}}, \mathbf{u})$.

In the γ-coupling domain with:

$g = g(\underline{\underline{\gamma}}) = \underline{\underline{\gamma}}^2/(1 + \underline{\underline{\gamma}})$ and $\underline{\underline{\beta}}^\alpha = \underline{\underline{\beta}}^1 = \underline{\underline{\beta}}$.

$$E^i{}_k = \begin{array}{c|cccc} i \backslash j \rightarrow & 0 & 1 & 2 & 3 \\ \hline 0 & \underline{\underline{\gamma}} & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 \\ 1 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_1 & 1 + g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_1\underline{\underline{\beta}}_3 \\ 2 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_1 & 1 + g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_2 & g\underline{\underline{\beta}}_2\underline{\underline{\beta}}_3 \\ 3 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}}_3 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_1 & g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_2 & 1 + g\underline{\underline{\beta}}_3\underline{\underline{\beta}}_3 \end{array}, \rightarrow \begin{array}{c|cccc} i \backslash j \rightarrow & 0 & 1 & 2 & 3 \\ \hline 0 & \underline{\underline{\gamma}} & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}} & 0 & 0 \\ 1 & -\underline{\underline{\gamma}}\,\underline{\underline{\beta}} & \underline{\underline{\gamma}} & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{array}. \tag{5.73}$$

And in the Γ-coupling domain with:

$g = g(\Gamma) = \Gamma^2/(1 + \Gamma)$ and and $u^\alpha = u^1 = u$.

$$E^i{}_k = \begin{array}{c|cccc} i \backslash j \rightarrow & 0 & 1 & 2 & 3 \\ \hline 0 & \Gamma^{-1} & \Gamma u_1 & \Gamma u_2 & \Gamma u_3 \\ 1 & 0 & 1 + g u_1{}^2 & g u_1 u_2 & g u_1 u_3 \\ 2 & 0 & g u_2 u_1 & 1 + g u_2{}^2 & g u_2 u_3 \\ 3 & 0 & g u_3 u_1 & g u_3 u_2 & 1 + g u_3{}^2 \end{array} \rightarrow \begin{array}{c|cccc} i \backslash j \rightarrow & 0 & 1 & 2 & 3 \\ \hline 0 & \Gamma^{-1} & \Gamma u & 0 & 0 \\ 1 & 0 & \Gamma & 0 & 0 \\ 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \end{array}. \tag{5.74}$$

E. A classical solution of the photon gravitational-mass/redshift problem.

a. The problem.

In accordance with both **w**-gauge theory, EM gravity and the perspective of Σ, any oscillator at a fixed position in a steady gravitational field will run slow by the factor $1/\ell \rightarrow 1/\Gamma = g_{00}^{½} = [1 - \mathbf{u}^2]^{½}$, causing its Σ−frequency to be reduced to $\upsilon = \upsilon_0/\Gamma$, where υ_0 is its standard, proper, rest, or w = 0, frequency. This Γ−expression of $\underline{\underline{w}}d^2$, and the γ−expression of $\underline{\underline{w}}d^2$ as well, were experimentally confirmed in early 1970 by means of Cesium-beam clocks.[30] And since the Σ−frequency of any kind of radiation must remain constant as it propagates throughout a steady medium,[31] the Cesium-beam clock experiments served to prove that the Γ−expression of $\underline{\underline{w}}d^2$ is sufficient to account for the gravitational redshift—solely in terms of the spatial constancy of the reduced frequency of the emitted photon, $\upsilon = \upsilon_e = \upsilon_0/\Gamma_e$.

Also, in accordance with the fundamental equation, $\lambda = h/p$, of quantum mechanics and the equally fundamental equation, $E = mc^2$, of special relativity theory and **w**-gauge theory, a photon has a purely kinetic energy and mass which are described in the γ−coupling domain by $E = h\upsilon = mc^2$.

Hence, the *photon gravitational-mass/redshift problem* is stated as follows:

> Given that the kinetic mass-energy of a photon is accurately expressed by $E = h\upsilon$, and that the Σ−frequency of a photon remains constant throughout a steady gravity field, how can we explain the photon's gravitational coupling, and thus the mass-energy that a photon *looses* in the course of propagating gravitationally uphill?

b. A solution.

The energy of a photon at a point where the gravitational potential is φ can be expressed equally well as $E_\varphi = hc/\lambda_\varphi$. And λ_φ is of course free to vary provided that its variation is linked to a variation in its intrinsic propagation speed, w^0_φ. We are thus at liberty to say, firstly, that the energy of a photon E is defined more generally by

$$E_\varphi = hc/\lambda_\varphi . \tag{5.27}$$

And secondly, we may require that $\lambda(\varphi) \equiv \lambda_\varphi$ must be consistent with both

$$\upsilon(\varphi) = \upsilon_\varphi = \upsilon_0/\Gamma_e = \text{constant}, \tag{5.28}$$

and the intrinsic propagation speed of the photon,

$$cw^0_\varphi = \upsilon_\varphi \lambda_\varphi. \tag{5.29}$$

Thirdly, from the EM gravity expression for p^i, the mass-energy of an excited atom which is at rest in the field is Γ times its rest value, E^0. Hence, the photon is emitted with an energy

$$E_e = \Gamma_e E^0 = \Gamma_e\, hc/\lambda_0 = hc/\lambda_e. \tag{5.30}$$

From these definitions we then obtain

$$\lambda_e = \lambda_0/\Gamma_e, \tag{5.31}$$

$$w^0_\varphi = g_{00} = 1/\Gamma^2, \tag{5.32}$$

$$\lambda_\varphi = \lambda_0\Gamma_e/\Gamma^2, \tag{5.33}$$

and finally,

$$E_\varphi = (h\upsilon_0)\Gamma^2/\Gamma_e. \tag{5.34}$$

In accordance with this scheme, the amount of energy that a photon looses in overcoming a potential Γ_e, with a constant Σ–frequency, is then given by

$$\Delta E = (h\upsilon_0)(\Gamma_e - 1/\Gamma_e), \tag{5.35a}$$

$$= 2\varphi_m\Gamma_e(h\upsilon_0) = 2\varphi_m E_e, \tag{5.35b}$$

where $\varphi_m = \frac{1}{2}\mathbf{u}^2 = -\,\Phi_n/c^2$.

F. The EM field-equations describing how the $g_{ik}(u^\alpha)$ potentials are sourced by the (1 + 3) energy-momentum tensor of mass-energy (exercises for the student).

1. The procedure

Using the $\Gamma^i_{jk}(u^\alpha)$ from Appendix A, evaluate the general form of the field equations of gravitation

$$f(u^\alpha,\varphi_m)_{jk} = \frac{\partial}{\partial x^i}\Gamma^i_{jk} + \Gamma^i_{jm}\Gamma^m_{ki} = -\,\kappa(T_{jk} - \tfrac{1}{2}g_{jk}T), \tag{5.36}$$

as given by Einstein[32] for the special (greatly simplified) case that $g = -\,1$, which is true for $g_{jk}(u^\alpha)$, to get EM field equations in terms of second order time and 3-space derivatives of u^α and φ_m.

2. Challenges for mathematicians.

Solve the EM field equations to get $u^\alpha(x^i)$ for the case that T_{ik} represents a spherically symmetric body at rest, so that the field is that of ideal Newtonian gravity. And use the equations, $\beta = \mathbf{u} + \mathbf{w}$, and $\beta^0 = \mathbf{u} + \mathbf{w}^0$, with and without $w^0 = (1 - \mathbf{u}^2)$, to calculate the angular precession of a planet's perihelion and the gravitational deflection of light. Optionally, use (5.42).

3. Some completeness Issues for the student of EM gravity.

Can you devise a more general definition of ds that puts $\nabla\times\mathbf{u}$ and/or $\nabla\bullet\mathbf{u}$ terms into g_{jk}?

Can you devise a thermodynamic derivation of the way that lightspeed should vary in the field, w^0_φ? And if so, can you incorporate that into a more general definition of ds that makes explicit use of w^0_φ, say by normalizing w to w^0_φ rather than c?

For experts in tensor analysis, Taub has constructed a very elegant Relativistic Fluid Mechanics[33] for the γ-coupling domain where g_{jk} (his $\eta_{\mu\nu}$) = $-G_{ik}$ = diag.(−1, 1, 1, 1). Consider revising his notation so that the signature of the fundamental tensor is diag.(1, −1, −1, −1), and then let the medium be the $\in_m$–continuum with $\eta_{\mu\nu} = g_{jk}(u^\alpha)$. Taub notes that in the limit where a mass-energy medium becomes incompressible, the speed of sound → the speed of light. Does that mean one could treat the $\in_m$–continuum as a medium that is effectively incompressible in the limit u → 0?

F. Some fruitful reinterpretations of quantum theory and the experimental data of high-energy particle physics.

To be completed.

VI. GFT3: The General Physical Principles and Field Equations of Hyper-Energy Theory.

A. Introduction

The 20th century disciplines of particle physics and cosmology uncovered six *profound structural and dynamical attributes* (PSADAs(1−6)) of Einstein's empty (${}^{0}\in_{m}$ = 0) 3−space. However, since Maxwell's energy rich $\in_{m}$–continuum (${}^{0}\in_{m}$ >> 0) supersedes empty 3−space, it follows that the PSADAs(1−6) of empty 3−space will be both more correctly and more profitably interpreted as six *profound structural and dynamical attributes* of Maxwell's ${}^{0}\in_{m}$–continuum.

It will be shown, for example, that given GFT1, PSADAs(1−2) of the ${}^{0}\in_{m}$–continuum are entirely sufficient to identify both the general (1 + p)-dimensional principles of *hyper energy theory* and the general type of field equations of *hyper-energy theory;* with *a priori* assurance that the theory is fully consistent with GFT1, GFT2, and the revolutionary 20th century findings of particle physics and cosmology. PSADAs(2−6) of the ${}^{0}\in_{m}$–continuum will then be seen to identify five structural and dynamical attributes of the ${}^{0}\in_{m}$–continuum that *hyper-energy theory* must account for. More precisely:

- PSADAs(1−2) will be shown to identify the general (1 + p)-dimensional physical principles of hyper-energy theory and the general (1 + p)-dimensional form (ψ_g) of its field equations; in terms of five *hyper-energy postulates* (HEPs(1−5)); thereby causing the (1 + 3)-dimensional properties of the ${}^{0}\in_{m}$–continuum (that were implied by GFT1) to be viewed as compactification solution of the ψ_g.
- PSADAs(2−6) will then be seen as five specific dynamic and structural attributes of the ${}^{0}\in_{m}$–continuum that hyper-energy theory must explain.

Sections VII − IX are then devoted to illustrating—with the help of some crucial *p-invariant solution characteristics*—hyper-energy theory's innate potential to explain PSADAs(2−6), and thus, by implication, everything else about this classically compactified and quantized universe.

B. PSADAs(1−2) and HEPs(1−5).

1. PSADA1 (Uncovered via the classically-grounded quantum theory of superstrings.)

The revelation of superstring theory that physical reality has a (1 + p)–dimensional parameterization with p = 9, has evolved over the last 90 years from the ground breaking publications by Gunnar Nordström (in 1914)[34] and Theodore Kaluza (in 1921)[35] which suggested that *classical electromagnetism and gravity* (CEMG) could be described as different aspects of a (1 + 4)-dimensional CEMG field. The history of the evolution of p = 4 to p = 25 and back down to p = 9 (driven by the desire to have the *weak* and the *strong* nucleon forces included in unified formulation), and the persistent absence of a suitable (1 + p)→'(1 + 3)'-dimensional *compactification theory*, are well documented. [36, 37, 38, 39, 40, 41]

Because superstring theory unifies a wealth of hard-earned data regarding $\delta\mathcal{E}_m$ particle transmutations and interactions, there seems to be little doubt that a (1 + p > 3)-dimensional reality and a significant fraction of superstring theory "*will survive in the final underlying laws*".[42]

We can therefore claim to know the following about the ${}^{0}\in_{m}$–continuum:

PSADA1: The ${}^{0}\in_{m}$–continuum of this universe is a (1 + p > 3)-dimensional energy structure that imposes a (1 + 3)-dimensional constraint on the dynamics and interactions of its $\delta\mathcal{E}_m$ particles via a *compactification physics* that is indigenous to hyper-energy theory.

Given GFT1 and PSADA1 we are naturally compelled to set down the following first postulate of hyper-energy theory:

HEP1: Maxwell's ${}^0\in_m$–continuum is a (1 + p > 3)-dimensional energy structure that is fabricated out of an elementary, (1 + p)-dimensional, non particulate, inviscid, and compressible *energy substance,* called *hyper-energy,* in such a way as to impose a (1 + 3)-dimensional constraint on the dynamics and interactions of its (1 + p)-dimensional $\delta\mathcal{E}_m$ particles.

2. PSADA2 (Uncovered via the cold, inflationary, expansion phase of big bang cosmology.)

In accordance with contemporary *big bang cosmology* (BBC) and HEP1, we can now claim to also know the following about the ${}^0\in_m$–continuum:

PSADA2: The ${}^0\in_m$–continuum of this universe came into being about 14 billion years ago, *entirely devoid of any* $\delta\mathcal{E}_m$ *particles* and initially expanding at a relatively high (so-called *inflationary*) rate, seemingly as a direct result of a single point-like explosion in the (1 + p)-dimensional continuum of hyper-energy.

Although HEP1 qualitatively answers the presently open BBC question of what existed before the big bang, it does not address the more fundamental open question of what caused the big bang. We lay down the following second postulate of hyper-energy theory in order to provide plausible answers to both of these questions:

HEP2: Prior to the big bang there was, and still is, a permanent (1 + p)-dimensional continuum of hyper-energy which is here postulated to possess a presently unknown form of internal energy (very likely vibrational) which has a natural tendency to produce stochastic eruptions that are referred to, for lack of a better terminology, as *percolations* or *percs* representing point-like explosions in the hyper-energy continuum.

We therefore propose that a hyper-energy *perc* provides a qualitative energy-conserving explanation for *a big bang*.[a] Accordingly we are led to lay down the third hyper-energy postulate as a partial answer to the question implied by HEP1; regarding the precise way in which the ${}^0\in_m$–continuum *was fabricated* from the hyper-energy continuum. Namely:

HEP3: The ${}^0\in_m$–continuum originated from one or more *percs* in the hyper-energy continuum.[a]

In Section VII the expansion dynamics of the ${}^0\in_m$–continuum is modeled in a way that depends on only *the fact of a perc* and not on *the detailed physical reason(s) for it*. Nevertheless, it might be heuristically useful to consider the following speculations on the physics underlying hyper-energy percs:

a. Non-critical heuristic speculations on perc physics.

It is reasonable to assume that the vibrational internal energy of hyper-energy sources the propagation of various types of energy waves which effectively endow the hyper-energy continuum with a potential, $\Phi[x^a(1 + p)]$,[b] for stochastically generating hyper-energy percs at points of convergent, constructive interference.

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[a] In Section VII, we will be explaining how (p – 3) percs could acount for a (1 + p)→'(1 + 3)'-dimensional compactification.
[b] The index (a) covers the integer range (0–p).

3. The general field equations of hyper-energy theory.

We are thus led to lay down the following fourth postulate of hyper-energy theory:

HEP4: The general field equations of hyper-energy theory (ψ_g) are, to a first order of approximation, a (p − 3)-dimensional extension of the ordinary (1 + 3)-dimensional, Lorentz covariant, nonlinear mathematics of *compressible fluid dynamics;* applied to the abstraction of a (1 + p)-dimensional continuum of inviscid, non particulate, hyper-energy; [43, 44, 45, 46] and encompassing the related but much newer disciplines of Soliton Theory and Chaos Theory.[47, 48, 49, 50]

4. This universe as a hierarchy of solutions (S_k) to specific k-sectors (ψ_k) of the ψ_g.

In concert with most field theories, we say that a *k–manifestation sector* of hyper-energy is defined by a *k-set* of *delimiting physical assumptions* (boundary conditions, sources, symmetries, parameter constraints, etc.) which transform ψ_g into a particular subset or *k-sector*, ψ_k, the various i^{th} solutions of which, S_{ki}, yield various *k–type manifestations of hyper-energy*. The k-index is generally alpha-numeric.

In view of HEPs(1−4) then, we are led to lay down the following fifth postulate of hyper-energy theory:

HEP5: The creation of the ${}^0\in_m$–continuum of this universe, its subsequent expansion dynamics, its internal $\delta\mathcal{E}_m$ field-particle evolution, and the interaction properties of all of its $\delta\mathcal{E}_m$ field-particles, can be mathematically described via a hierarchy of solutions to specific sectors of the ψ_g that are driven, at the lowest level of complexity and the highest level of potential, by one or more hyper-energy percs.

C. PSADAs(3−6) (As four additional specific challenges to the S_k of hyper-energy theory.)

1. PSADA3 (Uncovered via the Hubble expansion phase of BBC.)

Given the present BBC, PSADAs(1−2), and HEPs(1−5), we can now claim to also know the following about the ${}^0\in_m$–continuum:

PSADA3: The initial *inflationary* expansion rate of the ${}^0\in_m$–continuum spontaneously transitioned to a much slower and constant *Hubble rate* with a change in its internal structure that caused a fixed number of primordial Hydrogen, Helium, and Lithium atoms, and photons, to be precipitated.[a]

2. PSADA4 (Uncovered via theoretical investigations into the physical nature of time.)

In the spring of 1963, with support from the Air Force Office of Scientific Research, 22 leading physicists met at Cornel University for two days to brainstorm the question: W*hat is the physical nature of time?* Commenting on the outcome of the colloquium, Thomas Gold, the moderator of the colloquium, stated:

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[a] Manifesting a $\delta\mathcal{E}_m$ *particle temperature* of ≈ 4,000 K at a time of ≈ 100,000 years after the big bang**,** when the photons of the present 2.7 K *cosmic microwave background* (CMB) were first able to propagate relatively freely throughout the expanding ${}^0\in_m$–continuum. How this fixed supply of elemental atoms and photons became the complex universe of matter-energy and life-forms that exists today is a truly amazing story that has been summarized by many authors (see refs 36–39 and references sited therein).

“*We can not claim to have solved the main problems…and it may be that no very profound improvement in our understanding of time will ever take place; but it could also be that some new physical theory will be devised one day that depends on, or defines, a different concept of time…I personally think that our present understanding is inadequate and is perhaps holding us back from a better description of nature.*” [51]

This claim notwithstanding, the group did come up with a general qualification of time, and a corollary of it, which said *new physical theory* might be able to fully explain. As summarized by Bondi:

“If it is agreed that the expansion of the universe has a lot to do with the problem of time, a conclusion which I regard to virtually inescapable because this process leads to the dark night sky, the disequilibrium between matter and radiation, and to the fact that radiated energy is effectively lost, then we accept a very close connection between cosmology and the basic structure of our physics.” [52]

Hence, we can now claim to also know that:

PSADA4: The physical nature of time, and the structures and interactions of all $\delta\mathcal{E}_m$ particle, are all intimately related to the expansion of the $^0\in_m$–continuum of this universe.

3. PSADA5 (Uncovered via analyses of the elementary particle spectrum.)

The elementary particle spectrum encompasses both the long-life stable particles (and anti-particles) and the hundreds of mostly short-lived particles (and anti-particles) that 20th century science discovered can be created from the *kinetic mass-energy* unleashed in head-on collisions of *parent particles.* The end products of most high energy collision experiments are thus more *ordinary* (stable) $\delta\mathcal{E}_m$ *particles* than were present before the experiment.

From the seasoned perspective of Werner Heisenberg[53] this implies that a *fundamental theoretical understanding* of the particle spectrum must be possible, and along the same general, nonlinear, field-theoretic lines that so many other spectra have come to be understood. According to Heisenberg this implies that theorists should be earnestly seeking to formulate the following two things:

- A structure of the *so-called ‘empty space’* that provides an intrinsic *boundary condition* for the types of particles that can be transiently or permanently created from kinetic mass-energy.
- An underlying dynamics of matter.

These challenges, Heisenberg said, should be addressed *without any philosophical prejudices,* and the resulting theory *must be taken seriously*—provided that it rests on *well defined hypotheses that leave no essential points open.*

“*The particle spectrum can be understood only if the underlying dynamics of matter is known; dynamics is the central problem.*”[54]

We have already deduced that a formulation of the fluid dynamics of hyper-energy underlying the $^0\in_m$–continuum and $\delta\mathcal{E}_m$ particles *is central.* But now we can claim to also know the following about the $^0\in_m$–continuum:

PSADA5: The $^0\in_m$–continuum contains an intrinsic boundary condition that serves to delineate the various types of $\delta\mathcal{E}_m$ particles which can be formed from collisionally unleashed kinetic mass-energy.

4. PSADA6 (Uncovered via additional analyses of the elementary particle spectrum.)

From the seasoned perspective of elementary particle theorist, Hans Peter Dürr:

"It seems that here we come face to face with very general and most important relations that we had failed to take into consideration.

"If one of nature's fundamental symmetries is regularly found to be disturbed in the spectrum of elementary particles, the only possible explanation is that the universe, i.e., the substratum where the particles originated, is less symmetrical than the underlying physical law. It follows that there must be forces acting over long distances, or elementary particles of vanishing inertial mass.

"This is probably the best way of interpreting electrodynamics. Gravitation, too, could arise in this way, so that here we may hope to find a bridge to the principles on which Einstein wanted to base his unified field theory and cosmology…

"At present it looks very much as if we can interpret the whole of electrodynamics in terms of the asymmetry of the universe vis-à-vis the proton-neutron exchange or more generally vis-à-vis the isospin group." [55]

Hence, we can now claim to also know the following about the $^0\in_m$–continuum:

PSADA6: The $^0\in_m$–continuum contains a substratum which is characterized by a basic physical asymmetry in its (p − 3)-dimensions, in which both electromagnetism and gravity appear as *long range* pseudo (1 + 3)-dimensional *fields.*

D. Additional challenges for the S_k of hyper-energy theory.

In addition to PSADAs(2–6) of the $^0\in_m$–continuum, we have $E = mc^2$, $\underline{\underline{w}}d^{1-5}$, and the quantum physics of $\delta\mathcal{E}_m$ particles as additional challenges for the S_k of hyper-energy theory.

E. Valuable insights into the formal S_k of hyper-energy theory stemming from p-invariant symmetries and analogous sectors of ordinary hydrodynamics.

In Sections VII – IX, we exploit some symmetry-rooted, p-invariant, qualitative features of the S_k, and their well studied (1 + 3)-dimensional isomorphs, S'_k, to illuminate hyper-energy theory's innate ability for addressing all of the above described modeling challenges, via the following hierarchy of *hyper-energy solution sectors:* [a] [b]

- The *spherical hypershock* (ψ_{sh}, S_{sh}) sector of the ψ_g.
- The long range $\delta\mathcal{E}_m$-*particle structure* (ψ_{ps}, S_{ps}) sub-sector of S_{sh}.
- The massive and massless *soliton-*$\delta\mathcal{E}_m$*-particle propagation* (ψ_{spp}, S_{spp}) sub-sector of S_{sh}-S_{ps}.
- The massless *dispersion-*$\delta\mathcal{E}_m$*-wave propagation* (ψ_{dwp}, S_{dwp}) sub-sector of S_{sh}-S_{ps}.
- The *de-Broglie wave* (ψ_{dBw}, S_{dBw}) sub-sector of S_{sh}-S_{ps}.
- The *quantum behavior* (ψ_{qb}, S_{qb}) sub-sector of S_{sh}-S_{ps}-S_{dBw}.

[a] Thereby providing mathematicians with six reasonably well defined converging paths to the final field equations of hyper-energy theory and a qualitative understanding of some of the revolutionary technologies that may be then brought into being.

[b] As illustrated, we prime (′) all aspects of mass-energy fluid dynamics which are similar to hyper-energy fluid dynamics.

APPENDIX A.

THE CHRISTOFFEL FUNCTIONS (OF THE SECOND KIND) OF $g_{ik}(u^{\alpha})$

*******Derivations to be added before publication.**************

1. The Christoffel functions (of the second kind) of $g_{ik}(u^{\alpha})$ are defined by:

$$\Gamma^{i}_{jk} = \Gamma^{i}_{kj} = \Gamma^{i}_{jk}(\partial g_{jk}/\partial x^{i}) = \tfrac{1}{2}g^{im}[\partial g_{jm}/\partial x^{k} + \partial g_{mk}/\partial x^{j} - \partial g_{jk}/\partial x^{m}]. \quad \text{(A−1)}$$

With $g_{jk}(u^{\alpha})$ and $g^{jk}(u^{\alpha})$ given by (4.13), and $\varphi = \varphi_m = \tfrac{1}{2}\mathbf{u}^2$, and $\partial_k \equiv \partial/\partial x^k$, I obtained:

$$\Gamma^{0}_{00} = \mathbf{u}\bullet\nabla\varphi, \quad \text{(A−2)}$$

$$\Gamma^{0}_{0\alpha} = [-\nabla\varphi + \tfrac{1}{2}\mathbf{u}\times(\nabla\times\mathbf{u})]^{\alpha}, \quad \text{(A−3)}$$

$$\Gamma^{0}_{\alpha\beta} \equiv D_{\alpha\beta} = \tfrac{1}{2}(\partial_{\alpha}u^{\beta} + \partial_{\beta}u^{\alpha}), \quad \text{(A−4)}$$

$$\Gamma^{\gamma}_{00} = -[\nabla\varphi + \partial_0\mathbf{u} - \mathbf{u}(\mathbf{u}\bullet\nabla\varphi)]^{\gamma}, \quad \text{(A−5)}$$

$$\Gamma^{\gamma}_{0\beta} = -\tfrac{1}{2}u^{\gamma}[\nabla\varphi + (\mathbf{u}\bullet\nabla)\mathbf{u}]_{\beta} + \{\tfrac{1}{2}[\partial_{\gamma}u^{\beta} - \partial_{\beta}u^{\gamma}] \equiv C^{\gamma}{}_{\beta}\}, \quad \text{(A−6)}$$

$$\Gamma^{\gamma}_{\alpha\beta} = u^{\gamma}\{\tfrac{1}{2}[\partial_{\alpha}u^{\beta} + \partial_{\beta}u^{\alpha}] \equiv D_{\alpha\beta}\} = u^{\gamma}D_{\alpha\beta}, \quad \text{(A−7)}$$

Hence, $\Gamma^{k}_{\alpha\beta} = (1,u^{\gamma})D_{\alpha\beta} \equiv u^{k}D_{\alpha\beta}$. (A−8)

2. Subsidiary Formulas

In the process of deriving and implementing these components of Γ^{i}_{jk}, use was made of the following formulas:

$$(\mathbf{u}\bullet\nabla)\mathbf{u} = \nabla\varphi - \mathbf{u}\times(\nabla\times\mathbf{u}). \quad \text{(A−9)}$$

For any 3-vector, $\mathbf{A}$, $A^{\alpha}(2\,C^{\gamma}{}_{\alpha}) = [(\mathbf{A}\times(\nabla\times\mathbf{u})]^{\gamma}$. (A−10)

For any three 3-vectors $(\mathbf{a},\mathbf{b},\mathbf{c})$; $\mathbf{b}\bullet[(\mathbf{a}\bullet\nabla)\mathbf{c}] = \mathbf{a}\bullet[(\mathbf{b}\bullet\nabla)\mathbf{c}] + \mathbf{a}\bullet[\mathbf{b}\times(\nabla\times\mathbf{c})]$. (A−11)

Therefore, $\mathbf{a}\bullet[(\mathbf{u}\bullet\nabla)\mathbf{u}] + \mathbf{a}\bullet[\mathbf{u}\times(\nabla\times\mathbf{u})] = \mathbf{a}\bullet\nabla\varphi$, (A−12)

and $\mathbf{u}\bullet[(\mathbf{a}\bullet\nabla)\mathbf{u}] = \mathbf{a}\bullet\nabla\varphi$. (A−13)

REFERENCES AND NOTES

[24] In the 20 years following Einstein's passing, the inevitable correctness of his nonlinear classical logic was being championed by the likes of (Dirac, 1963), (Dürr, 1965), and (Heisenberg, 1976).

[25] This is the path (mass-energy$\rightarrow\delta\in_m\rightarrow{}^0\in_m$) through which ${}^0\in_m$ was deduced from $E = mc^2$ in (Var, 1975).

[26] R. Wrede, *Introduction to vector and tensor analysis,* pp. 1–7 (Dover, 1972).

[27] R. Wrede, op. cit., p. 184.

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[29] This raises an important general question concerning tensor analysis in general and (1 + 3) tensor analysis, in particular, which I will simply state and not attempt to explore in any great detail, here. Namely, what are the physical significances of various *covariant* (1 + 3) quantities? Or, more generally—regarding any physics that is being described—how much extra information is made available by dividing the physics into its *contravariant* and *covariant* forms? Because it has a lot to do with the physics of ${}^0\in_m$ and its 100% coupling of mass-energy structures to ${}^0\in_m$, I would venture to say, a great deal.

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